\[ \theta_1 = 0, 0.01, \ldots, \frac{\pi}{2} \]  
\[ Z_{IP}(\theta_1) = 376.7 \cdot \cos(\theta_1) \]  
\[ Z_{IS}(\theta_1) = \frac{376.7}{1 \cdot \cos(\theta_1)} \]  
\[ \theta_2(\theta_1) = \arcsin\left(\frac{\sin(\theta_1)}{1.7}\right) \]  
\[ Z_{IP}(\theta_1) = \frac{376.7 \cdot \cos(\theta_2(\theta_1))}{1.7} \]  
\[ Z_{IS}(\theta_1) = \frac{376.7}{1.7 \cdot \cos(\theta_2(\theta_1))} \]  
\[ pP(\theta_1) = \frac{Z_{IP}(\theta_1) - Z_{IP}(\theta_1)}{Z_{IP}(\theta_1) + Z_{IP}(\theta_1)} \]  
\[ pS(\theta_1) = \frac{Z_{IS}(\theta_1) - Z_{IS}(\theta_1)}{Z_{IS}(\theta_1) + Z_{IS}(\theta_1)} \]  
\[ \theta_B = \arctan(1.7) \]  
\[ \theta_B = 0 \]  
\[ RS(\theta_B) = 0.236 \]  
S-wave reflectance at Brewster's angle
Brewster's angle is clearly visible as the place where the P-wave reflectance = 0.
\[ n_2 = 1.75 \quad \lambda_0 = 514.4 \cdot 10^{-9} \]

\[ n_1 = 1 \]

\[ i = 1, 2 \ldots 100 \]

\[ \theta_{1_i} = (i - 1) \frac{\pi}{2.99} \quad \text{Range of incident angles} \]

\[ \theta_{2_i} = \arcsin \left( \frac{\sin(\theta_{1_i})}{n_2} \right) \quad \text{Reflected angles} \]

\[ \theta_{3_i} = \theta_{1_i} \]

\[ Z_0 = 376.7 \quad \text{Impedance of free space} \]

\[ Z_1 = \frac{Z_0}{n_1} \quad Z_2 = \frac{Z_0}{n_2} \quad Z_3 = \frac{Z_0}{n_1} \]

\[ k_{2d} = 2 \pi \quad \text{One wavelength slab, wavelength does not matter} \]

\[ k_{2d_i} = k_{2d} \cos(\theta_{2_i}) \quad \text{takes into account effective thickness of slab at different angles} \]

**Effective impedances for P waves**

\[ Z_{11P_i} = Z_1 \cos(\theta_{1_i}) \quad Z_{21P_i} = Z_2 \cos(\theta_{2_i}) \quad Z_{31P_i} = Z_3 \cos(\theta_{3_i}) \]

**Effective impedances for S waves**

\[ Z_{11S_i} = \frac{Z_1}{\cos(\theta_{1_i})} \quad Z_{21S_i} = \frac{Z_2}{\cos(\theta_{2_i})} \quad Z_{31S_i} = \frac{Z_3}{\cos(\theta_{3_i})} \]

**Transformed impedance for P waves**

\[ Z_{311P_i} = \frac{Z_{31P_i} \cos(k_{2d_i}) + i \cdot Z_{21P_i} \sin(k_{2d_i})}{Z_{21P_i} \cos(k_{2d_i}) + i \cdot Z_{31P_i} \sin(k_{2d_i})} \]

**Transformed impedance for S waves**

\[ Z_{311S_i} = \frac{Z_{31S_i} \cos(k_{2d_i}) + i \cdot Z_{21S_i} \sin(k_{2d_i})}{Z_{21S_i} \cos(k_{2d_i}) + i \cdot Z_{31S_i} \sin(k_{2d_i})} \]

\[ \rho_{P_i} = \frac{Z_{311P_i} - Z_{11P_i}}{Z_{311P_i} + Z_{11P_i}} \quad \text{reflection coefficient for P waves} \]
\[ TP_i = 1 - (|p_{P_i}|^2) \quad \text{Transmittance for P waves} \]

\[ TS_i = 1 - (|p_{S_i}|^2) \quad \text{Transmittance for S waves} \]

Transmittance of one wavelength thick slab

\[ \text{Angle of incidence in radians} \]
\[ n_2 = 1.60 \]
\[ n_1 = 1 \]
\[ i = 1, 2, \ldots, 10000 \quad \text{Use lots of points to show detail in final graphs} \]
\[ \theta_{1i} = (i - 1) \cdot \frac{x}{2.9999} \quad \text{Range of incident angles} \]
\[ \theta_{2i} = \sin^{-1} \left( \frac{\sin(\theta_{1i})}{n_2} \right) \quad \text{Reflected angles} \]
\[ \theta_{3i} = \theta_{1i} \]
\[ Z_0 = 376.7 \quad \text{Impedance of free space} \]
\[ Z_1 = \frac{Z_0}{n_1} \quad Z_2 = \frac{Z_0}{n_2} \quad Z_3 = \frac{Z_0}{n_1} \]
\[ \lambda_0 = 1.06 \times 10^{-6} \quad \text{Wavelength in free space} \]
\[ d_2 = 0.075 \times 10^{-3} \quad \text{Thickness of slab} \]
\[ k_2 = 2 \pi \frac{n_2}{\lambda_0} \quad \text{Wavevector in slab} \]
\[ k_{2d} = k_2 \cdot d_2 \]
\[ k_{2d_i} = k_{2d} \cdot \cos(\theta_{2i}) \quad \text{takes into account effective thickness of slab at different angles} \]

**Effective impedances for P waves**

\[ Z_{11P_i} = Z_1 \cdot \cos(\theta_{1i}) \]
\[ Z_{21P_i} = Z_2 \cdot \cos(\theta_{2i}) \]
\[ Z_{31P_i} = Z_3 \cdot \cos(\theta_{3i}) \]

**Effective impedances for S waves**

\[ Z_{11S_i} = \frac{Z_1}{\cos(\theta_{1i})} \]
\[ Z_{21S_i} = \frac{Z_2}{\cos(\theta_{2i})} \]
\[ Z_{31S_i} = \frac{Z_3}{\cos(\theta_{3i})} \]

**Transformed impedance for P waves**

\[ Z_{311P_i} = \frac{Z_{31P_i} \cdot \cos(k_{2d_i}) + i \cdot Z_{21P_i} \cdot \sin(k_{2d_i})}{Z_{21P_i} \cdot \cos(k_{2d_i}) + i \cdot Z_{31P_i} \cdot \sin(k_{2d_i})} \]
\[ Z_{311S_i} = \frac{Z_{311S_i} \cdot \cos(k2d_i) + i \cdot Z_{211S_i} \cdot \sin(k2d_i)}{Z_{211S_i} \cdot \cos(k2d_i) + i \cdot Z_{311S_i} \cdot \sin(k2d_i)} \]

\[ \rho^P_i = \frac{Z_{311P_i} - Z_{111P_i}}{Z_{311P_i} + Z_{111P_i}} \quad \text{reflection coefficient for P waves} \]

\[ \rho^S_i = \frac{Z_{311S_i} - Z_{111S_i}}{Z_{311S_i} + Z_{111S_i}} \quad \text{reflection coefficient for S waves} \]

\[ TP_i = 1 - \left( |\rho^P_i| \right)^2 \quad \text{Transmittance for P waves} \]

\[ TS_i = 1 - \left( |\rho^S_i| \right)^2 \quad \text{Transmittance for S waves} \]

Transmittance for P Waves

Angle of incidence in radians

The transmittance demonstrates that the slab acts as a "comb" filter

Note that the transmittance is one at Brewster's angle


\[ n_2 = 3.5 \]
\[ n_1 = 1 \]
\[ i = 1, 2 \ldots 10000 \]

Use lots of points to show detail in final graphs

\[ Z_0 = 376.7 \quad \text{Impedance of free space} \]

\[ Z_1 = \frac{Z_0}{n_1} \quad Z_2 = \frac{Z_0}{n_2} \quad Z_3 = \frac{Z_0}{n_1} \]

\[ \lambda_0 = 1319 \times 10^{-9} \quad \text{Wavelength in free space} \]

\[ k_2 = 2 \pi \frac{n_2}{\lambda_0} \quad \text{Wavevector in slab} \]

\[ d_{2i} = 5 \times 10^{-6} \frac{i - 1}{9999} \]

\[ k_2 d_{2i} = k_2 d_2_i \]

Effective impedances for P waves

\[ Z_{11P_i} = Z_1 \quad Z_{21P_i} = Z_2 \quad Z_{31P_i} = Z_3 \]

Effective impedances for S waves are the same

\[ Z_{11S_i} = Z_1 \quad Z_{21S_i} = Z_2 \quad Z_{31S_i} = Z_3 \]

Transformed impedance for P waves

\[ Z_{311P_i} = \frac{Z_{31P_i} \cos(k_2 d_{2i}) + i \cdot Z_{21P_i} \sin(k_2 d_{2i})}{Z_{21P_i} \cos(k_2 d_{2i}) + i \cdot Z_{31P_i} \sin(k_2 d_{2i})} \]

Transformed impedance for S waves is the same in normal incidence

\[ Z_{311S_i} = \frac{Z_{31S_i} \cos(k_2 d_{2i}) + i \cdot Z_{21S_i} \sin(k_2 d_{2i})}{Z_{21S_i} \cos(k_2 d_{2i}) + i \cdot Z_{31S_i} \sin(k_2 d_{2i})} \]

\[ \rho_{P_i} = \frac{Z_{311P_i} - Z_{11P_i}}{Z_{311P_i} + Z_{11P_i}} \quad \text{reflection coefficient for P waves} \]

\[ \rho_{S_i} = \frac{Z_{311S_i} - Z_{11S_i}}{Z_{311S_i} + Z_{11S_i}} \quad \text{reflection coefficient for S waves is the same as for P waves in normal incidence} \]

\[ T_{P_i} = 1 - (|\rho_{P_i}|)^2 \quad \text{Transmittance for P waves} \]

\[ T_{S_i} = 1 - (|\rho_{S_i}|)^2 \quad \text{Transmittance for S waves is the same as for P waves} \]
The transmittance demonstrates that the slab acts as a "comb" filter.
n2 := 1.55 \quad n1 := 1 \quad \text{refractive indices}

Z0 := 376.7 \quad \text{Impedance of free space}

\begin{align*}
Z1 &= \frac{Z0}{n1} \\
Z2 &= \frac{Z0}{n2} \\
Z3 &= \frac{Z0}{n1}
\end{align*}

\lambda_0 := 1060 \cdot 10^{-9} \quad \text{Wavelength in free space}

k2 := 2 \cdot \frac{\pi \cdot n2}{\lambda_0} \quad \text{Wavevector in slab}

\theta_1 := 30 \cdot 2 \cdot \frac{\pi}{360} \quad \text{Angle of incidence in radians}

\theta_2 := \arcsin \left( \frac{\sin(\theta_1)}{n2} \right) \quad \text{Angle of refraction}

\theta_2 = 0.32845462 \quad \text{radians}

For input coherent light the two faces of the slab are not independent and we need to use the transformed impedance concept

d2 := 3.4 \cdot 10^{-3} \cdot \cos(\theta_2) \quad \text{Effective thickness of slab}

k2d := k2 \cdot d2

\frac{k2d}{2 \cdot \pi} = 4.705921 \cdot 10^3 \quad \text{Reduce by an integral multiple of } 2\pi \quad k2d := k2d - 4705.2 \cdot \pi \quad k2d = 5.78678629

\textbf{Effective impedances for } P \textbf{ waves}

Z11P := Z1 \cdot \cos(\theta_1) \quad Z21P := Z2 \cdot \cos(\theta_2) \quad Z31P := Z3 \cdot \cos(\theta_1)

\textbf{Effective impedances for } S \textbf{ waves}

Z11S := \frac{Z1}{\cos(\theta_1)} \quad Z21S := \frac{Z2}{\cos(\theta_2)} \quad Z31S := \frac{Z3}{\cos(\theta_1)}

\text{Transformed impedance for } P \text{ waves}

Z311P := Z21P \cdot \frac{Z31P \cdot \cos(k2d) + i \cdot Z21P \cdot \sin(k2d)}{Z21P \cdot \cos(k2d) + i \cdot Z31P \cdot \sin(k2d)}

\text{Transformed impedance for } S \text{ waves}

Z311S := Z21S \cdot \frac{Z31S \cdot \cos(k2d) + i \cdot Z21S \cdot \sin(k2d)}{Z21S \cdot \cos(k2d) + i \cdot Z31S \cdot \sin(k2d)}

\rho_P := \frac{Z311P - Z11P}{Z311P + Z11P} \quad \text{reflection coefficient for } P \text{ waves}
\[ p_S := \frac{Z_{311S} - Z_{11S}}{Z_{311S} + Z_{11S}} \quad \text{reflection coefficient for S waves} \]
\[ TP := 1 - (|\rho_P|)^2 \quad \text{Transmittance for P waves} \]
\[ TS := 1 - (|\rho_S|)^2 \quad \text{Transmittance for S waves} \]
\[ TP = 0.9719794 \quad \text{Transmittance for P waves} \]
\[ TS = 0.93537237 \quad \text{Transmittance for S waves} \]

For an input wave made up of 50% P and 50% S the overall transmittance is
\[ T := 0.5 \cdot TP + 0.5 \cdot TS \]
\[ T = 0.95367588 \quad \text{Overall transmittance is 95.4\%} \]

(c) For equal amounts of P and S linear polarization is at 45 degrees to P direction

After reflection:
\[ \rho_P = -0.08344372 + 0.14511287i \]
\[ \rho_S = -0.13374809 + 0.21619223i \]
\[ |\rho_P| = 0.16739355 \]
\[ |\rho_S| = 0.25421965 \]
\[ \arg(\rho_P) = 2.09265039 \quad \text{radians} \]
\[ \arg(\rho_S) = 2.12481889 \]
\[ (\arg(\rho_S) - \arg(\rho_P)) = 0.03216849 \quad \text{radians} \]
\[ \frac{\arg(\rho_S) - \arg(\rho_P)}{\deg} = 1.84311891 \quad \text{degrees} \]

This is elliptically polarized light, as shown below
\[ \omega t := 0, 0.001 \ldots 2 \pi \quad \text{variable to plot ellipse} \]

We can write
\[ EP(\omega t) = \text{Re} \left( \rho_P e^{i \omega t} \right) \]
\[ ES(\omega t) = \text{Re} \left( \rho_S e^{i \omega t} \right) \]
(d) For 30% S, 70% P

\[ \theta_P := \text{atan} \left( \frac{\sqrt{30}}{\sqrt{70}} \right) \]

\[ \theta_P = 33.21091076 \text{ deg} \]

Initial angle with P direction

\[ T_{\text{overall}} = 0.3 \cdot TS + 0.7 \cdot TP \]

\[ T_{\text{overall}} = 0.96099729 \]

\( \rho_P = -0.08344372 + 0.14511287i \)

\( \rho_S = -0.13374809 + 0.21619223i \)

\[ |\rho_P| = 0.16739355 \]

\[ |\rho_S| = 0.25421965 \]

\[ \text{arg}(\rho_P) = 2.09265039 \text{ radians} \]

\[ \text{arg}(\rho_S) = 2.12481889 \]

\[ (\text{arg}(\rho_S) - \text{arg}(\rho_P)) = 0.03216849 \text{ radians} \]

This is elliptically polarized light, as shown below

\( \omega t = 0, 0.001 .. 2 \pi \) variable to plot ellipse

We can write

\[ \text{EP}(\omega t) := \text{Re} \left( \rho_P e^{i \cdot \omega t} \right) / 0.7 \]

\[ \text{ES}(\omega t) := \text{Re} \left( \rho_S e^{i \cdot \omega t} \right) / 0.3 \]

\[ \frac{\text{arg}(\rho_P)}{\text{deg}} = 119.90003558 \]

\[ \frac{\text{arg}(\rho_S)}{\text{deg}} = 121.74315448 \]
(6) \[ \rho P1 = \frac{Z21P - Z11P}{Z21P + Z11P} \]
\[ \rho S1 = \frac{Z21S - Z11S}{Z21S + Z11S} \]
\[ R P1 = (|\rho P1|)^2 \]
\[ R S1 = (|\rho S1|)^2 \]
\[ T P1 = 1 - R P1 \]
\[ T S1 = 1 - R S1 \]
\[ T P1 = 0.97009807 \quad T P1^2 = 0.94109026 \]
\[ T S1 = 0.93362209 \quad T S1^2 = 0.87165021 \]