Interaction of Service Providers in Task Delegation Under Simple Payment Rules

Richard J. La and Jeonghoon Mo

***Abstract*— We consider a scenario where an application service provider (ASP) hires a network service provider (NSP) to deliver its service and pays for the employment of the NSP. We study the interaction between these two providers under simple payment rules as a Stackelberg game. We first show, under the assumption that the ASP knows the true utility function of the NSP, the existence of a unique equilibrium of the game and investigate its properties when the NSP is risk averse. Then, we relax the assumption that the ASP is aware of the NSP's true utility function and point out a potential source of difficulty in designing a pricing mechanism that encourages truth-telling by the NSP.**

I. INTRODUCTION

The issue of network pricing has attracted much attention in the past decade and has been studied in several different contexts. Designing suitable network pricing mechanisms is important for both recovering the cost of providing existing network services and encouraging deployment of new services and expansion of network capacities. Here we only provide a short list of studies on network pricing. MacKie-Mason and Varian studied the problem of pricing congestible network resources and the effects of different pricing schemes on performance and industry structure [12]. Kelly in his seminal paper [6] suggested an optimization framework for rate allocation in the Internet [6]. Based on his framework, he and his colleagues proposed usage based pricing, where prices of resources depend on their congestion level [7]. He and Walrand [5] and Shakkottai and Srikant [15] investigated the pricing between multiple Internet service providers.

In this paper we study the interaction between an Application Service Provider (ASP) and a Network Service Provider (NSP), which we call *players*: We assume that the ASP wants to offer a network application service. However, the ASP does not have a means of directly bringing the service to the customers. Instead, it has to employ an NSP to provide the service on its behalf. The NSP in return receives a payment for its service according to a *contract* they both agree to in advance. For example, consider a Location-Based Service (LBS) provider that wants to provide a list of local businesses of interest, advertisements (e.g., notification of a sale at a nearby business) or electronic coupons (e-coupons) to customers in the coverage area of a cellular NSP. If the LBS provider is not the cellular NSP itself, it may need to rely on the cellular NSP to track the locations of the subscribers and deliver the service to them.

Before we can design a suitable contract between these two players, we first need to understand their behavior, which is the focus of this paper. We assume that the ASP first offers a contract to the NSP, and the NSP either accepts it or rejects it. If the NSP accepts the contract, it chooses the amount of resource it will expend providing the service, called its *effort*, and gets paid according to the *output* observed by both players. The output depends on two factors: The first one is the effort made by the NSP. The second is beyond the control of the NSP and is assumed unknown to both the ASP and the NSP when the contract is presented. We model it using a random variable.

Both the ASP and the NSP are assumed selfish; the ASP is risk neutral and is only interested in maximizing its expected profit given by the revenue it collects from the service delivered to the customers minus the payment to the NSP. Similarly, the NSP wants to maximize its expected utility, which is a function of its profit given by the difference between the payment from the ASP and the cost of providing the service. The cost is a function of the effort made by the NSP providing the service.

Since both the ASP and the NSP are assumed selfish, we borrow the tools from game theory to investigate their interaction. Game theory has been applied successfully in recent years to modeling and examining the relations between multiple selfish, non-cooperative entities in networking contexts (e.g., [9], [13], [14]). In particular, we model the interaction as a *Stackelberg game* [4], where the ASP is the *leader* and the NSP is the *follower*. A similar formulation has been utilized by Korilis et al. for a routing problem [8] and by Basar and Srikant for a revenue maximization problem with a single service provider and many adaptive users [1]. Since the type of the NSP is not known to the ASP when the game is played, i.e., when the ASP offers a contract, the game is of *incomplete information* [4].

We only consider a family of affine contracts by the ASP, under which the payment to the NSP consists of (i) a fixed amount and (ii) a performance based payment proportional to the output. We first show the existence of a unique equilibrium (Theorem 1 in Section IV). Then, we investigate how the degree of risk aversion by the NSP, which is captured by its utility function and known to the ASP, affects the choice of the optimal contract for the ASP at the unique equilibrium. Our study reveals several interesting properties that are consistent with the intuition (Theorem 2 in Section IV) and points at a potential key source of

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R. J. La is the Department of Electrical & Computer Engineering, University of Maryland, College Park, MD 20742, USA hyongla@isr.umd.edu

J. Mo is with the Department of Information and Industrial Engineering, Yonsei University, Seoul, Korea j.mo@yonsei.ac.kr

difficulty for designing a pricing scheme with the *incentive compatibility* property (Theorem 3 in Section IV) [11].

The rest of the paper is organized as follows: Section II describes the setup of our problem and assumptions we introduce for our analysis. Section III explains the Stackelberg model we use for modeling the interaction between the ASP and the NSP. Our main results are presented in Section IV. We conclude in Section V.

II. MODEL AND SET-UP

A. Application service provider and network service provider

In our problem there are two players – an ASP and an NSP. The ASP needs to hire an NSP to bring its service to subscribers and offers a contract ξ to the NSP, which determines the payment to the NSP if accepted. The NSP either accepts or rejects the offered contract without any negotiation. If the NSP accepts the offered contract, it decides the amount of resource it wants to expend (called its *effort*) providing the service on behalf of the ASP, and receives a payment from the ASP according to the contract, based on the *output* produced by the service and observed by the both players.

The ASP collects revenue from the provided service. The revenue is a function of the output, which is assumed to depend on two factors: (i) The effort E by the NSP, and (ii) a random variable (rv) V. As mentioned earlier, the effort E reflects the amount of resource expended by the NSP providing the service. The rv V represents any *unknown* factor that affects the output, and is beyond the control of the NSP. Its value is unknown to both the ASP and the NSP at the time the contract is offered, and only its distribution $\mathcal{G}$ is known to them. We assume that the rv V is continuous and has a compact range $S_V := [v_{\min},\ v_{\max}]$. Although the value of rv V is assumed unknown to the NSP when the NSP is offered the contract, we assume that the NSP can observe its value (over time), whereas the ASP cannot directly observe it.

We denote the output by $Z(V, E)$ or simply by Z when the dependence on V and E is clear. When the output is Z, ASP's revenue is $R(Z)$. The payment from the ASP to the NSP computed according to the contract ξ, is denoted by $P_\xi(Z)$. The profit or payoff of the ASP is then given by $R(Z) - P_\xi(Z) =: A_\xi(Z)$.

The NSP incurs a cost for providing the service, which depends on its effort E. We denote this cost by $C(E)$, which satisfies the following conditions[1]: (i) $C(0) = 0$, and (ii) $C : \mathbb{R}_+ \to \mathbb{R}_+$ is strictly increasing, where $\mathbb{R}_+ := [0, \infty)$. Since the NSP collects a payment of $P_\xi(Z)$ from the ASP, NSP's profit or payoff is given by $P_\xi(Z) - C(E) =: N_\xi(Z, E)$.

The overall efficiency of the partnership between these two players is measured by the total revenue collected by the ASP minus the cost of delivering the service incurred by the NSP, i.e., $R(Z) - C(E)$. Note that this also equals the sum of the profit of the ASP, $A_\xi(Z)$, and the profit of the NSP, $N_\xi(Z, E)$.

B. Cost, output, payment, and revenue

In this paper we consider the following scenarios.

[A1] We assume that the revenue is an affine function of the output, i.e., $R(Z) = \theta + \phi \cdot Z$ for some $\phi > 0$ and $\theta \geq 0$. The fixed component, θ, of the revenue may represent, for example, the fees the ASP charges to its customers for accessing the service (e.g., monthly subscription fees). Since the constant θ does not affect the choice of the ASP (as will be clear), without loss of generality, for the purpose of analysis we assume $\theta = 0$ unless stated otherwise.

[A2] We assume that the payment between the two players is an affine function of the output: The contract ξ is given by a pair (α, β), $\alpha \in \mathbb{R}$, $0 \leq \beta \leq \phi$, and the set of contracts under consideration is given by $\Xi = \{(\alpha, \beta) \mid \alpha \in \mathbb{R},\ 0 \leq \beta \leq \phi\}$. When the output is z, ASP's payment to the NSP is given by

$$P_\xi(z) = \alpha + \beta \cdot z\ . \tag{1}$$

In other words, the payment consists of (i) a fixed amount, α, and (ii) the performance based payment, $\beta \cdot z$. The parameter β determines how much of the output based revenue (i.e., $\phi \cdot z$) the ASP will share with the NSP as an incentive to increase the output.[2] Obviously, other forms of contracts are possible. However, it turns out that this family of affine contracts we consider is general enough to capture some of key expected behaviors displayed by the players, as will be shown in Section IV.

[A3] When the NSP chooses effort $E = e$, its cost (for the effort) is given by

$$C(e) = c \cdot e \quad \text{for some } c > 0\ . \tag{2}$$

This is a reasonable assumption as the cost incurred by the NSP while delivering the service will indeed increase proportionally to the resource it spends. The property we require for our analysis is that $C(e)$ is a convex function of the effort e (although it need not be strictly convex). As mentioned earlier, $C(E)$ denotes only the cost of the effort E. We will discuss shortly how other costs incurred by the NSP but not directly related to its effort (e.g., an initial investment or administrative cost) can be addressed.

[A4] The output, which is a function of the effort E and the rv V, is given by

$$Z(v, e) = \lambda\ (1 - \exp(-v \cdot e)) \quad \text{for some } \lambda > 0. \tag{3}$$

Even though a more general output function can be used to obtain similar results, we employ this form of output in order to facilitate the analysis and to provide some insight. An important property required for our analysis and captured by this function, which one would expect to hold in practice, is that the output is a *concave* function of the effort.

[1] Here $C(E)$ denotes the cost of the effort E. We will discuss how other cost (e.g., cost incurred once for configuring the network to offer the service) may be recovered in Section III.

[2] Since ASP's revenue per output is ϕ, a rational ASP would not pay more than ϕ per output to the NSP.

In order to motivate these assumptions, let us consider the earlier example of an LBS provider and a cellular NSP. It is reasonable to assume that the amount of resource expended by the cellular NSP (e.g., the number of timeslots used in a time-division multiple access system) is proportional to the number of advertisements or e-coupons, which we simply refer to as *listings*, it delivers to the subscribers.

Suppose that a customer is offered e listings of, for example, restaurants nearby. The customer is interested in each restaurant with probability p, independently of others. When the customer is interested in at least one restaurant, it selects one of them. In this case, the probability that the customer will not choose any of the restaurants is given by $(1-p)^e$, while the probability of picking one of the advertised restaurants is $1-(1-p)^e$. If we take $\mathbf{Pr}$[customer selects a listing] to be the output per customer and the arrival rate of the customers is λ, it is reasonable to model the output as

$$Z(e) = \lambda\ (1-(1-p)^e) = \lambda(1-\exp(-v \cdot e))\ ,$$

where $v = -\ln(1-p)$. However, in practice the exact value of p (hence, the value of the parameter v) is unlikely to be known in advance and will clearly be beyond the control of the cellular NSP.

Note that the ASP's revenue (with the assumption $\theta = 0$ in place) is given by $R(Z(v,e)) = \phi \cdot \lambda(1-\exp(-v \cdot e))$ and the payment to NSP is $P_\xi(Z(v,e)) = \alpha + \beta \cdot \lambda(1-\exp(-v \cdot e))$. It is clear that, without loss of generality, we can assume $\phi = 1$ by scaling up λ by ϕ and normalizing β by ϕ. Therefore, in the rest of the paper we assume $\phi = 1$ and $\beta \in [0, 1]$. In this case, β denotes the *fraction* of the output based revenue the ASP is willing to share with the NSP as an incentive.

Under these assumptions, given a contract $\xi = (\alpha, \beta)$, when $V = v$ and $E = e$, the ASP's payment to the NSP is

$$P_\xi(Z(v,e)) = \alpha + \beta \cdot \lambda(1-\exp(-v \cdot e))\ . \qquad (4)$$

Consequently, the profit of the ASP is

$$A_\xi(Z(v,e)) = (1-\beta)\ \lambda\ (1-\exp(-v \cdot e)) - \alpha\ ,$$

and the profit of the NSP is

$$N_\xi(Z(v,e), e) = \alpha + \beta \cdot \lambda(1-\exp(-v \cdot e)) - c \cdot e\ . \qquad (5)$$

III. STACKELBERG GAME MODEL

We assume that both the ASP and the NSP are selfish and are interested in maximizing their individual (expected) utility. This is a reasonable assumption for the following reason: Once the contract is signed and the value of rv V is estimated, the NSP is likely to select an optimal effort that will maximize its payoff. This is because the output, hence the profit, of the NSP depends only on its selected effort E once it has the estimate of the value of rv V. The ASP, when aware of this behavior by the NSP, should also try to choose the contract ξ that will maximize its own profit. However, since the ASP does not know the value of rv V and is given only its distribution $\mathcal{G}$ at the time of selecting the contract, the ASP ought to maximize its *expected* profit. Recall that the ASP picks the contract offered to the NSP first, who decides whether to accept the contract or not, and then its effort if it accepts the contract. Hence, this problem can be naturally formulated as a Stackelberg game with the ASP as the leader and the NSP as the follower.

a) Risk neutrality and risk aversion: In our setting, the ASP is assumed *risk neutral*[3] and its expected utility is given by its expected profit. The NSP, on the other hand, may be *risk averse*.[4] We model this using a utility function U; when NSP's profit is ω, the utility it receives is given by $U(\omega)$. The utility function U reflects the degree of risk aversion by the NSP. In particular, we use Arrow-Pratt measure of Relative Risk Aversion (RRA) [16, p.189] to quantify it: The Arrow-Pratt measure of RRA of a player with a utility function $U(\omega)$ at ω is defined to be

$$RRA(\omega) = -\frac{\omega \times U''(\omega)}{U'(\omega)}\ . \qquad (6)$$

If the utility function is increasing and concave, then the RRA is non-negative. In this case, the larger RRA is, the more risk averse the player is.

Suppose that the utility function of the NSP is of the form

$$U_a(\omega) = (\omega)^a\ ,\ 0 < a \leq 1\ . \qquad (7)$$

This family of utility functions are known as *isoelastic* utility functions [3]. For these utility functions, the RRA is given by

$$RRA(\omega) = -\frac{\omega \cdot a \cdot (a-1) \cdot \omega^{a-2}}{a \cdot \omega^{a-1}} = 1 - a\ , \qquad (8)$$

which is constant regardless of the value of ω. Note from (8) that (i) the degree of risk aversion decreases with utility function parameter a, and (ii) the case $a = 1$, i.e., linear utility function, corresponds to a risk neutral player with the RRA equal to zero. In this paper we use these isoelastic utility functions to model a range of risk aversion by the NSP and investigate how it affects ASP's choice of contract to offer to the NSP.

b) Optimal effort by NSP: Suppose that the utility function is strictly increasing and differentiable (e.g., isoelastic utility function in (7)). Once a contract $\xi = (\alpha, \beta)$ is accepted and the value of rv V is estimated to be v by the NSP, the optimal effort $e^\star := e^\star(\xi, v)$ that maximizes the profit, hence the utility, of the NSP satisfies the following necessary and sufficient Kuhn-Tucker (KT) condition [2]: For every $v \in S_V$,

$$\begin{aligned} &\frac{\partial}{\partial e} N_\xi(Z(v,e), e)\bigg|_{e=e^\star} \\ &= \frac{\partial}{\partial e} P_\xi(Z(v,e))\bigg|_{e=e^\star} - \frac{d}{de} C(e)\bigg|_{e=e^\star} \\ &= \beta \cdot \frac{\partial}{\partial e} Z(v,e)\bigg|_{e=e^\star} - c \end{aligned}$$

[3] A player that cares only about the *expected* payoff is said to be *risk neutral*.

[4] A player is *risk averse* when the utility of its expected payoff is larger than the expected utility from the payoff [16, pp.177-178].

$$= \beta \cdot \lambda \cdot v \cdot \exp(-v \cdot e)\Big|_{e=e^\star} - c$$
$$\begin{cases} = 0 & \text{if } e^\star > 0, \\ \leq 0 & \text{if } e^\star = 0. \end{cases}$$

From the above condition, we obtain

$$e^\star(\xi, v) = \begin{cases} \frac{1}{v} \ln(\lambda \cdot v \cdot \beta / c) & \text{if } \lambda \cdot v \cdot \beta > c, \\ 0 & \text{if } \lambda \cdot v \cdot \beta \leq c. \end{cases} \tag{9}$$

Note that the optimal effort $e^\star(\xi, v)$ does *not* depend on either the fixed amount α in the contract or the utility function parameter a. This is a consequence of the following observation: From the assumption that the utility function is strictly increasing in profit, the marginal utility (i.e., the derivative of the utility function with respect to the profit) is strictly positive. Therefore, in order to maximize its utility, if the optimal effort is strictly positive, the NSP should increase its effort until the marginal payment from the ASP equals its marginal cost, i.e., the derivative of the *profit* is equal to zero. However, it is clear from (5) that the derivative of NSP's profit depends on neither α nor a. This observation, although rather intuitive, has interesting and serious consequences on the effectiveness or suitability of affine contracts as we will illustrate in Section IV.

Since the optimal effort by the NSP does not depend on the fixed payment α, with a little abuse of notation, we denote it by $e^\star(\beta, v)$ hereafter, omitting the fixed payment α in the contract ξ. When the NSP applies the optimal effort, the output is given by

$$Z(v, e^\star(\beta, v)) = \begin{cases} \lambda - \frac{c}{\beta \cdot v} & \text{if } \lambda \cdot v \cdot \beta > c, \\ 0 & \text{if } \lambda \cdot v \cdot \beta \leq c. \end{cases}$$

c) Expected utilities and the optimization problem of ASP: As mentioned earlier, when designing the contract to present to the NSP, the ASP should take into account the selfish behavior of the NSP. Given a contract $\xi = (\alpha, \beta)$, under the assumed selfish behavior the *expected utility* of the NSP is given by

$$\begin{aligned} &\mathbf{E}\left[U_a(\alpha, \beta)\right] \\ &= \mathbf{E}\left[U_a(P_\xi(Z(V, e^\star(\beta, V))) - C(e^\star(\beta, V)))\right] \qquad (10) \\ &= \int \Big(\alpha + \beta \cdot Z(v, e^\star(\beta, v)) - c \cdot e^\star(\beta, v)\Big)^a d\mathcal{G}(v) \\ &= \int \Big(\alpha + \mathbf{1}\left\{c < \lambda \cdot v \cdot \beta\right\} \Big((\lambda \cdot \beta - c/v) \\ &\qquad - c \cdot \ln(\lambda \cdot v \cdot \beta / c)/v\Big)\Big)^a d\mathcal{G}(v) \\ &=: \mathcal{N}_\xi^a \ . \end{aligned}$$

Here, given a contract (α, β) and under the assumed selfish behavior by the NSP, we denote the expected utility of the ASP $\mathbf{E}\left[U_a(N_\xi(Z(V, e^\star(\beta, V)), e^\star(\beta, V)))\right]$ by $\mathbf{E}\left[U_a(\alpha, \beta)\right]$ for notational simplicity.

Suppose that the ASP is aware of the NSP's utility function parameter a. We will consider the case where this assumption does not hold in Section IV. Being aware of the NSP's strategy, the goal of the ASP is to find an optimal contract $\xi^\star = (\alpha^\star, \beta^\star) \in \Xi$ that maximizes its expected utility, which is equal to its expected profit from the assumption of risk neutrality and is given by

$$\begin{aligned} \mathbf{E}\left[A_\xi(Z)\right] &= \int A_\xi(Z(v, e^\star(\beta, v)))\ d\mathcal{G}(v) \\ &= \int \Big((1-\beta) Z(v, e^\star(\beta, v)) - \alpha\Big)\ d\mathcal{G}(v) \\ &=: \mathcal{A}_\xi \ . \end{aligned} \tag{11}$$

To be more precise, given the utility function U_a of the NSP, the ASP wants to solve the following optimization problem:

$$\text{maximize}_{\xi \in \Xi} \qquad \mathcal{A}_\xi \tag{12}$$
$$\text{subject to} \qquad \mathcal{N}_\xi^a \geq U_{a,\min} \tag{13}$$

where $U_{a,\min}$ is the *reserve utility* of the NSP. In other words, the NSP does not accept any contract offered by the ASP that will result in the expected utility smaller than its reserve utility $U_{a,\min}$. The reserve utility is assumed to depend on any initial cost or other administrative/operational expenses that the NSP bears to provide the service on behalf of the ASP, but does not depend on its effort. We denote such cost by Λ, and the reserve utility is given by $U_{a,\min} = U_a(\Lambda)$.

Since we can scale both the payment and the revenue by Λ (by scaling α, c and λ), without loss of generality, we assume that $\Lambda = 1$, hence $U_{a,\min} = 1 \equiv U_{\min}$ for all $a \in (0, 1]$. For each fixed $a \in (0, 1]$, we define

$$\mathcal{C}_a := \{\xi \in \Xi \mid \mathcal{N}_\xi^a \geq 1\} \ ,$$

which is the set of contracts acceptable to the NSP with utility function parameter a. The constrained optimization problem in (12) - (13) can then be rewritten in the following simpler form:

$$\text{maximize}_{\xi \in \mathcal{C}_a} \qquad \mathcal{A}_\xi \tag{14}$$

The ASP can simplify its problem in (14) further, based on the following observation. After a closer look at (14) (or (12) - (13)) one can show that, at an optimal contract $\xi^\star$, the constraint on $\mathcal{N}_\xi^a$ will be active, i.e., $\mathcal{N}_{\xi^\star}^a = 1$. This is because, for a fixed β, the expected profit $\mathcal{A}_\xi$ of the ASP given by (11) is strictly decreasing in α, whereas $\mathcal{N}_\xi^a$ is strictly increasing in α. Therefore, if the constraint is not active, the ASP can decrease α till the constraint is just satisfied in order to increase its own profit. In fact, by the same argument, as the performance based payment (i.e., $\beta \cdot Z$) does not depend on α, for every fixed $\beta \in [0, 1]$, there exists a unique $\alpha^\star$ such that $\mathcal{N}_{\xi=(\alpha^\star, \beta)}^a = 1$, which depends on the utility function parameter a. This suggests that we can think of the smallest $\alpha^\star$ that satisfies the constraint in (13) as a function of β and a, i.e., $\alpha^\star : [0, 1] \times (0, 1] \to \mathbb{R}$. Therefore, in the case of affine payments, the following much simpler problem can be considered instead:

$$\text{maximize}_{\beta \in [0,1]} \qquad \mathcal{A}_{(\alpha^\star(\beta, a), \beta)} \tag{15}$$

It is clear that, for all $a \in (0, 1]$, when $\beta = 0$,

$$\int (\alpha^\star(0, a))^a\, d\mathcal{G}(v) = (\alpha^\star(0, a))^a = 1$$

or $\alpha^\star(0, a) = 1$, i.e., the fixed payment $\alpha^\star(0, a)$ equals the reserve utility of the NSP as the output will be zero and there will be no performance based payment.

IV. MAIN RESULTS

In our Stackelberg game delineated in the previous section, given a fixed utility function of the NSP, the action space of the ASP is given by the set $\mathcal{C}_a$. Similarly, the action space of the NSP is the set of mappings $\mathcal{E}_a := \{e : \mathcal{C}_a \times S_V \to \mathbb{R}_+\}$, which determine the effort given the contract $\xi \in \mathcal{C}_a$ and the value of rv V. However, it is clear that the strategy of a rational NSP will be given by (9). Thus, an equilibrium of the game is given by a pair $(\xi^\star, e^\star)$, where $e^\star$ is defined through (9) and $\xi^\star$ is an optimal contract in $\mathcal{C}_a$ that maximizes the expected profit of the ASP in (11), given the strategy $e^\star$ of the NSP. As argued in the previous section, $\xi^\star$ is given by $(\alpha^\star(\beta^\star, a), \beta^\star)$ for some $\beta^\star \in [0, 1]$.

We first introduce the following assumptions.

Assumption 1: (i) The probability $\mathbf{P}[V > c/\lambda]$ is strictly positive. (ii) For every feasible contract $(\alpha, \beta) \in \mathcal{C}_a$, the profit of the NSP $\alpha + \beta \cdot Z(v, e^\star(v, \beta)) - c \cdot e^\star(v, \beta)$ is strictly positive for all $v \in S_V$.

The first assumption ensures that, for some $a \in (0, 1]$ (for example, $a = 1$), there exists a non-degenerate equilibrium with $\beta^\star > 0$. It also implies that $v_{\max} > c/\lambda$. The second assumption, although not necessary, is introduced to simplify the proofs of our main results (Theorems 1 - 3 below). A trivial sufficient condition is that $\alpha > 0$.

Our first result states that there exists a unique equilibrium of the Stackelberg game for all $a \in (0, 1]$.

Theorem *1:* For every utility function parameter $a \in (0, 1]$, (i) $\alpha^\star(\beta, a)$ is a decreasing function of β, and (ii) there exists a unique solution $\beta^\star(a)$ to the optimization problem in (15).

We denote the unique solution to (15) at the equilibrium by $\beta^\star(a)$, and the corresponding expected profit of the ASP by $\mathcal{A}^\star(a)$. The following theorem proves the monotonicity properties of $\alpha^\star(\beta^\star(a), a), \beta^\star(a)$ and the expected profit $\mathcal{A}^\star(a)$ of the ASP at the equilibrium with respect to the utility function parameter a.

Theorem *2:* (i) The solution $\beta^\star(a)$ (resp. $\alpha^\star(\beta^\star(a), a)$) is increasing (resp. decreasing) in utility function parameter a. (ii) $\lim_{a\uparrow 1} \beta^\star(a) = 1$.[5]
(iii) ASP's expected profit $\mathcal{A}^\star(a)$ increases with a.

Theorem 2(i) states that as the NSP becomes less risk averse (with increasing a), it becomes more willing to take the risk of receiving a smaller profit and its objective gets more aligned with that of the ASP, which is assumed risk neutral. This allows the ASP to increase the performance based payment, which encourages the NSP to raise its effort, while reducing the fixed payment, α, at the same time. The higher output produced by the raised effort by the NSP in turn leads to a larger profit for the ASP (Theorem 2(iii)). Finally, when the NSP becomes risk neutral, i.e., $a = 1$, this results in $\beta^\star = 1$ and the NSP's objective is perfectly aligned with that of the ASP (Theorem 2(ii)). In this case the ASP charges a fixed fee of $\mathbf{E}[Z] - \Lambda$ to the NSP (i.e., $\alpha^\star(1, 1) = \Lambda - \mathbf{E}[Z]$) for the right to provide the service on behalf of the ASP, which can be viewed as a franchise fee. Recall that Z is the output (which is equal to the output based revenue) and Λ is any initial or other operating cost not directly related to the effort. The NSP on the other hand pays the franchise fee and collects all of the output based revenue $R(Z) = Z$ of the ASP for the service it provides.

[5]When the revenue per output ϕ is not one, this limit equals ϕ.

Theorem *3:* (i) Let $a_1, a_2 \in (0, 1]$ such that $a_1 < a_2$ and $\xi^\dagger = (\alpha^\dagger, \beta^\dagger) := (\alpha^\star(\beta^\star(a_1), a_1), \beta^\star(a_1))$. If $\mathbf{P}\left[V > c/(\lambda \cdot \beta^\dagger)\right] > 0$, we have

$$U_{\min} = \mathbf{E}\left[U_{a_1}(\alpha^\dagger, \beta^\dagger)\right] < \mathbf{E}\left[U_{a_2}(\alpha^\dagger, \beta^\dagger)\right] \ . \qquad (16)$$

(ii) For every $v \in S_V$, the efficiency of the system given by the revenue $R(Z(v, e^\star(\beta^\star(a), v)))$ of the ASP minus the cost $C(e^\star(\beta^\star(a), v))$ of the NSP at the equilibrium is non-decreasing in a.

Note that Theorem 3(i) states that the equilibrium contract $\xi^\dagger = (\alpha^\dagger, \beta^\dagger)$ of the ASP for an NSP with utility function parameter a_1 will be acceptable to any NSP with a larger utility function parameter a_2. This result follows from the intuition that, if an NSP finds a contract ξ acceptable, so should another NSP that is less risk averse, and is a simple consequence of Jensen's inequality [10, p.111].

This intuitive observation has the following important implication for the design of a pricing mechanism. Recall that in our analysis we assumed that the ASP knows the value of NSP's utility function parameter a. Suppose that this is no longer true, and the ASP does *not* know the NSP's utility function. Then, Theorem 3(i) implies that the NSP has an incentive to *lie* about its utility function parameter and to pretend that it is smaller than its true value in an attempt to increase its expected utility.[6]

An even more interesting observation is that if the NSP decides to lie about its utility function, it is virtually impossible for the ASP to detect it. The reason for this is that, as mentioned in Section III, given an acceptable contract ξ, the optimal effort $e^\star(\xi, v)$ in (9) does *not* depend on the utility function parameter a and the NSP would behave in exactly the same way no matter what the value of a is. As a result, the ASP cannot prevent the NSP from lying about its utility function.

These observations result in following consequences:

(i) If the NSP accepts the contract offered by the ASP based on an incorrect utility function, the overall efficiency, measured as the total revenue raised from offering the service minus the cost of providing the service, suffers (Theorem 3(ii)).

[6]The NSP can increase its expected utility by lying about its utility function. This is because if it revealed its true utility function to the ASP, at the equilibrium its expected utility would be equal to its reserve utility (which is one) as mentioned at the end of Section III.

(ii) Deployment of new applications can be hindered because ASPs may not find it profitable enough to offer the services based on the incorrect estimate of expected profits when NSPs lie about their true utility.

These findings suggest that simple contracts (such as the affine contracts studied in this paper) may not be suitable in practice; the overall efficiency can suffer as a consequence of a failure to provide the NSP with an incentive to reveal its true utility function. This observation can also be viewed as a result of inefficient sharing of the profits between the ASP and the NSP under the affine contracts, which fails to provide the NSP with an incentive to be truthful. These clearly point out some of difficulties in designing a good pricing mechanism for the task delegation problem between selfish entities in the presence of asymmetry of information.

V. CONCLUSION

We investigated the interaction between an ASP and an NSP where the ASP is interested in securing the employment of the NSP to offer its service. We considered a family of affine contracts that determine the payment by the ASP to the NSP as a function of the output observed by the both parties. First, the problem is formulated as a Stackelberg game between the two players with the ASP as the leader and the NSP as the follower under the assumption that the ASP knows the utility function of the NSP. We established the existence of a unique equilibrium. We also demonstrated several interesting and intuitive properties of the equilibrium when the NSP is allowed to be risk averse. Second, when the ASP is unaware of the true utility of the NSP, our findings suggest that the NSP may have an incentive to lie about its utility function to increase its expected utility, possibly revealing one of main sources of difficulty in the problem of pricing mechanism design for task delegation between selfish entities.

REFERENCES

[1] T. Basar and R. Srikant, "A Stackelberg network game with a large number of followers," *Journal of Optimization Theory and Applications*, vol. 115(3), pp. 479-490, Dec. 2002.

[2] D. P. Bertsekas, *Nonlinear Programming*, Athena Scientific, Belmont (MA), 1995.

[3] O. J. Blanchard and S. Fischer, *Lectures on Macroeconomics*, MIT Press, Cambridge (MA), 1989.

[4] D. Fudenberg and J. Tirole, *Game Theory*, MIT Press, Cambridge (MA), 1991.

[5] L. He and J. Walrand, "Pricing and revenue sharing strategies for Internet service providers," *IEEE Journal on Selected Areas in Communications*, vol. 24(5), pp. 942-951, May 2006.

[6] F. Kelly, "Charging and rate control for elastic traffic," *European Trans. on Telecommunications*, vol. 8(1), pp. 33-37, Jan. 1997.

[7] F. Kelly, A. Maulloo and D. Tan, "Rate control for communication networks: shadow prices, proportional fairness and stability," *Journal of the Operational Research Society*, vol. 49(3), pp. 237-252, Mar. 1998.

[8] Y. A. Korilis, A. A. Lazar and A. Orda, "Achieving network optima using Stackelberg routing strategies," *IEEE/ACM Trans. on Networking*, vol. 5(1), pp. 161-173, Feb. 1997.

[9] R. J. La and V. Anantharam, "Optimal routing control: repeated game approach," *IEEE Trans. on. Automatic Control*, vol. 47(3), pp. 437-450, Mar. 2002.

[10] K. Lange, *Optimization*, Springer Texts in Statistics, Springer-Verlag (NY), 2004.

[11] J.-J Laffont and D. Martimort, *The Theory of Incentives: The Principal-Agent Model*, Princeton University Press, Princeton (NJ), 2002.

[12] J. K. Mackie-Mason and H. R. Varian, "Pricing congestible network resources," *IEEE Journal on Selected Areas in Communications*, vol. 13(7), pp. 1141-1149, Sep. 1995.

[13] P. Marbach, "Priority service and max-min fairness," *IEEE/ACM Trans. on Networking*, vol. 11(5), pp. 733-746, Oct. 2003.

[14] A. Orda, R Rom and N. Shimkin, "Competitive routing in multi-user communication networks," *IEEE/ACM Trans. on Networking*, vol. 1(5), pp. 510-521, Oct. 1993.

[15] S. Shakkottai and R. Srikant, "Economics of network pricing with multiple ISPs," *IEEE/ACM Trans. on Networking*, vol. 14(6), pp. 1233-1245, Dec. 2006.

[16] H. R. Varian, *Microeconomic Analysis*, 3rd ed., Norton, New York (NY), 1992.