ENEE312H Spring 2002 Solutions to Homework Set 1

Problem 1:

a) Arsenic (As) is in column V of the periodic table and thus is a donor in silicon (Si), and boron (B), in column III, is an acceptor in Si. We thus have: $N_d = 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{15}$ cm^{-3}, so $N_d - N_a$ is positive and the sample is n-type.

b) $n_0 = N_D = N_d - N_a = 5 \times 10^{15}$ cm^{-3}.

c) $p_0 = \dfrac{n_i^2}{n_0} = 10^{20}/5 \times 10^{15} = 2 \times 10^4$ cm^{-3}.

d) Aluminum (Al), from column III, is an acceptor in Si, and antimony (Sb), from column V, is a donor in Si. We thus have: $N_d = 10^{16}$ cm^{-3} and $N_a = 10^{17}$ cm^{-3}, so $N_d - N_a$ is negative and the sample is p-type with $p_0 = N_A = N_a - N_d = 9 \times 10^{16}$ cm^{-3} and $n_0 = \dfrac{n_i^2}{p_0} = 10^{20}/9 \times 10^{16} = 1.1 \times 10^3$ cm^{-3}.

e) Gallium (Ga) and boron (B) are both from column III and thus are acceptors in Si. We thus have: $N_d = 0$ and $N_a = 10^{15} + 5 \times 10^{15} = 6 \times 10^{15}$ cm^{-3}, so $N_d - N_a$ is negative and the sample is p-type with $p_0 = N_A = N_a - N_d = 6 \times 10^{15}$ cm^{-3} and $n_0 = \dfrac{n_i^2}{p_0} = 10^{20}/6 \times 10^{15} = 1.7 \times 10^4$ cm^{-3}.

Problem 2:
For purposes of discussion assume that we are dealing with an n-type sample. Since we don't know how far away from intrinsic we will be, we must begin with the exact expressions

$$n_0 = \frac{N_D}{2} + \frac{N_D}{2}\sqrt{1 + \frac{4n_i^2}{N_D^2}}$$

$$p_0 = -\frac{N_D}{2} + \frac{N_D}{2}\sqrt{1 + \frac{4n_i^2}{N_D^2}}$$

where $N_D = N_d - N_a$. To keep things general we say we want p_0 to be less than some fraction a of n_0, i.e., $p_0 < a\, n_0$. Thus we have

$$\frac{p_0}{n_0} = \frac{-\dfrac{N_D}{2} + \dfrac{N_D}{2}\sqrt{1 + \dfrac{4n_i^2}{N_D^2}}}{\dfrac{N_D}{2} + \dfrac{N_D}{2}\sqrt{1 + \dfrac{4n_i^2}{N_D^2}}} < a$$

which reduces to

$$-\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{n_i^2}{N_D^2}} < \frac{a}{2} + a\sqrt{\frac{1}{4} + \frac{n_i^2}{N_D^2}}$$

and then becomes

$$(1-a)\sqrt{\frac{1}{4} + \frac{n_i^2}{N_D^2}} < \frac{1+a}{2}$$

Dividing by $(1-a)$, squaring both sides, moving the 1/4 to the right, and taking the square root, we have finally

$$\frac{n_i}{N_D} < \sqrt{\frac{(1+a)^2}{4(1-a)^2} - \frac{1}{4}}$$

We want to evaluate this first when a is 0.1 and then when a is 0.01. In the first case we find that we must have $n_i/N_D < 0.35$, or since what the question asked for was N_D/n_i, $N_D/n_i > 2.8$. In the second case we find that we must have $N_D/n_i > 9.9$.

Problem 3:
  a) $\lambda(AlSb) =$ 0.76 μm Opaque, blocks the entire visible spectrum.
  b) $\lambda(GaP) =$ 0.55 μm Looks orange b/c it transmits yellow and red wavelengths.
  c) $\lambda(ZnS) =$ 0.34 μm Looks clear because it transmits the entire visible spectrum.
  d) $\lambda(InAs) =$ 3.75 μm Opaque.
  e) $\lambda(Si) =$ 1.10 μm Opaque.

Problem 4:
$n_0 = N_D = N_d - N_a = 5 \times 10^{15}$ cm^{-3}
$p_0 = n_i^2/n_0 = 10^{20}/5 \times 10^{15} = 2 \times 10^4$ cm^{-3}
$V = 1 \ \mu m^3 = 10^{-12}$ cm^3
$e = n_0 V = 5 \times 10^{15} \times 10^{-12} = 5000$ electrons
$h = p_0 V = 2 \times 10^4 \times 10^{-12} = 2 \times 10^{-8}$ holes < 1 hole

Problem 5:
The unit cell of Si is a cube of side length 5.43Å, so the volume of the unit cell is $(5.43Å)^3 = (5.43 \times 10^{-8}$ cm$)^3 = 1.60 \times 10^{-22}$ cm^3. In order to compute the atomic density, we must determine how many atoms should be assigned to the unit cell. The eight corner atoms are shared among the eight cells touching the corner and therefore contribute one atom to the unit cell. The six atoms on the faces of the cube are shared with adjacent cells and therefore contribute three atoms. Four atoms lie entirely within the unit cell, so the total number of atoms per unit cell is eight.
The atomic density of crystalline silicon, in atoms per cm^3, is $8/1.60 \times 10^{-22}$ cm^3 = 5×10^{22} cm^{-3}.

Research question:
In 1913, Neils Bohr formulated a theory of the atomic structure of atoms to explain the wavelength spectrum observed empirically from atomic hydrogen. He made two important assumptions: (a) that stable orbits about the nucleus exist where it is possible for the electron to reside without radiating, and (b) that when disturbed, such an electron may remain in the original orbit, transition to another stable orbit, or leave the influence of the nucleus entirely. The atom would therefore radiate or absorb amounts of energy equal to the energy difference between initial and final states.

Bohr supposed that the electron moves in a circle (for simplicity), and that it is maintained in a stable orbit by an equilibrium between electrostatic attraction and centrifugal force. The model predicts the wavelength spectrum of hydrogen if the stationary states have angular momentum that

is an exact multiple of $\hbar = h/2\pi$, where h is Planck's constant. The energy level of the Nth stationary state is

$$E_N = -\frac{mZ^2 q^4}{2\hbar^2 N^2}$$

where m is the mass of the electron, Z is the nuclear charge, q is the elementary charge, and zero energy is when the electron is at infinity (removed from the nucleus). Unfortunately, the Bohr model with planetary orbits produces correct results only when a single electron is involved – that is, only for atomic Hydrogen.