Computation of magnetic fields from recording surfaces with multiple tracks

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Calculations of the magnetic fields emanating from a recorded surface that contains an infinite number of identical tracks are presented. The solutions are obtained by assuming that the magnetization is uniform through the thickness of the film and can be represented by Fourier series in the other two dimensions. Solutions are obtained for both longitudinal and vertical recording. To make these calculations useful Fourier distributions are obtained with arctan transitions that are both representative and easy to use. These magnetic fields are then used to calculate the voltage from an idealized reproduce head. Effects on the output voltage caused by track width, track separation, head registration, and transition lengths can then be calculated. Examples are given and it is shown that the effects can be large, compared to the usual infinite track width calculations, when the track width or the track separation become comparable to the recorded wavelength.

I. INTRODUCTION

In the seminal article by Wallace, the magnetic fields from a recording surface were calculated assuming an infinite track width. Since then there have been numerous articles in the literature that have examined the effects of finite track widths, but they have always been for isolated tracks. To our knowledge, the problem has never been attacked in its entirety, for multiple tracks.

In order to determine the effects of finite track widths, we have calculated the magnetic fields from a recorded surface containing an infinite number of identical tracks. Once the magnetic fields are determined, measurable quantities such as the voltage from a magnetic recording head can be calculated. We have performed numerous such calculations and the results can be compared to the same quantities calculated for an infinite track width. For the read head voltages we show the effects of track widths and track separations on the output voltages. We show that these effects can become significant when the wavelength of the signal approaches the width of the track.

II. THEORY

The magnetic medium is a thin film of thickness \( \delta \) in the \( z \) direction and infinite extent in the \( x \) and \( y \) directions. We assume that the magnetization is uniform through the thickness of the medium and can be represented by Fourier series in the other two dimensions. The magnetic field \( H \) in the absence of any currents, can be written as the gradient of a scalar potential, \( \Phi = -\nabla \Phi \). The scalar potential must satisfy the equation \( \nabla \Phi = 4\pi \mu M \), where \( M \) is the magnetization in the thin-film medium. This latter equation comes from the constitutive equation \( B = H + 4\pi M \), and the Maxwell's equation, \( \nabla \cdot B = 0 \). We have performed the calculations for the more realistic constitutive equation, \( B = \mu H + 4\pi M \), but for the sake of simplicity we will present here the case where \( \mu = 1 \) in the thin-film medium.

The first magnetization pattern we will consider is in the \( x \) direction. This would correspond to longitudinal recording and can be written as

\[
M_x = M_x \sum_{n,m} I_{n,m} \sin k_x n \cos k_y m,
\]

where \( M_x \) is the saturation magnetization, \( I_{n,m} \) are the normalized Fourier coefficients, and \( k_x \) and \( k_y \) are the wave numbers in the \( x \) and \( y \) directions (\( k_x = 2\pi n/\lambda_x \), \( k_y = 2\pi m/\lambda_y \)) while \( \lambda_x \) and \( \lambda_y \) are the wavelengths. This representation is for a magnetization that is symmetric and repetitive. We could, of course, further generalize this by including phase angles for a nonsymmetric case or Fourier integrals for a nonrepetitive case.

We will obtain solutions for \( \Phi \) in three regions: (I) above the film; (II) in the film; and (III) below the film. The three solutions are

(I) \( \Phi^{(I)} = \sum_{n,m} \Phi_{n,m}^{(I)} e^{-kz} \cos k_x x \cos k_y y, \)

(II) \( \Phi^{(II)} = \sum_{n,m} \left( \Phi_{n,m}^{(II)} e^{-kz} + \Phi_{n,m}^{(II)} e^{kz} \right) \frac{4\pi M I_{n,m} k_x}{k^2} \cos k_x x \cos k_y y, \)

(III) \( \Phi^{(III)} = \sum_{n,m} \Phi_{n,m}^{(III)} e^{kz} \cos k_x x \cos k_y y, \)

where \( k = \sqrt{k_x^2 + k_y^2} \). These solutions were obtained by using: (i) the symmetry of the problem; (ii) the fact that \( \Phi \) must vanish at \( z = \pm \infty \); (iii) the method of separation of variables; and (iv) the fact that the trigonometric functions form a complete orthogonal set.

The coefficients in Eqs. (2)–(4) may be determined by requiring the continuity of the normal \( B \) and the transverse \( H \) at the surfaces of the film \((z=0,-\delta)\). The results are
\[ \phi_{n,m}^{(I)} = -2\pi M_s L_{n,m}(1 - e^{-k\delta})k_x/k^2, \quad (5) \]
\[ \phi_{n,m}^{(II)} = 2\pi M_s L_{n,m}e^{k\delta}k_x/k^2, \quad (6) \]
\[ \phi_{n,m}^{(III)} = 2\pi M_s L_{n,m}k_x/k^2, \quad (7) \]
\[ \phi_{n,m}^{(IV)} = 2\pi M_s L_{n,m}(1 - e^{k\delta})k_x/k^2, \quad (8) \]

where again we have used the orthogonality of the trigonometric functions. The magnetic fields can now be completely determined by putting the coefficients from Eqs. (5)-(8) into the solutions, Eqs. (2)-(4), and calculating the magnetic-field components.

For perpendicular recording the magnetization will be in the z direction, and can be written as

\[ M_z = -M_s \sum_{n,m} P_{n,m} \cos k_x x \cos k_y y. \quad (9) \]

The solutions will be the same as for the longitudinal case except for the last term in Eq. (3) which will now vanish since \( \nabla \cdot M = 0 \) in all three regions. The coefficients can now be calculated as before, with the result for the (I) region given by

\[ \phi_{n,m}^{(I)} = -2\pi M_s P_{n,m}(1 - e^{-k\delta})/k. \quad (10) \]

We have chosen the relative phases for the magnetization distributions in Eqs. (1) and (9) to be the same as Wallace. The fields that we have calculated reduce to those found by Wallace, who used completely different techniques, in the limit of one Fourier component \((k_x \rightarrow k, \ k_y \rightarrow 0)\). Wallace showed that the fields above the medium are identical for longitudinal and vertical recording. Our results show that this is no longer true if \( k_y \neq 0 \).

Wallace also calculated the fields inside a high-permeability head by using the method of images. In our case, this would correspond to adding a fourth region above the recording medium where \( B = \mu H \) and \( \mu \) is the permeability of the head. We would then have to add an exponentially increasing solution in the region between the head and the film and then proceed as before. The modifications are the same as those found by Wallace and, in particular, the field inside the head is found to be the same as the field previously found above the film except multiplied by the factor \( 2\mu/(\mu + 1) \).

III. FOURIER DISTRIBUTIONS

For these techniques to be useful, we must find Fourier series that are both representative and easy to use. To this end, we have made an extensive study of numerous Fourier series and have found that the most useful ones are series with arctan transitions where \( a \) is the transition length. The Fourier series that we use can be shown to be identical to the series that would be constructed from the linear superposition principle. The Fourier coefficients that would correspond to a magnetization variation in the x direction are given by

\[ X_a = 4e^{-2\pi a/(2n+1)}\lambda/\pi(2n+1). \quad (11) \]

\[ Y_m = \frac{2e^{-[2\pi\mu/(W+b)]}}{\pi m} W+b, \quad m>0, \]
\[ Y_m = \frac{W}{W+b}, \quad m=0, \quad (12) \]

where \( W \) is the track width, \( b \) is the track separation, and \( a \) is the arctan transition length in the y direction. The coefficients in Eq. (1) would now be simply \( L_{n,m} = X_a Y_m \). The coefficients for vertical recording would be constructed similarly. With the use of Eqs. (11) and (12) we are now in the position to calculate the magnetic-field components.

IV. MAGNETIC-FIELD CALCULATIONS

Examples of the calculated field components are shown in Fig. 1. These fields were calculated for the case of longitudinal recording with the following values: \( M_s = 1, z = 0, \lambda = 20, W = 20, b = 10, \delta = 0.1, \) and \( a = 1 \) (arbitrary units).
These values were chosen for illustration. The profiles of the $H_x$ and the $H_y$ fields along the center of the tracks are similar to those found in the literature for single pulses, but now we have a complete description of the fields everywhere, including the space between the tracks. The $H_y$ field is new.

V. HEAD VOLTAGE

We now calculate the voltage from an idealized reproducing head by finding the time rate of change of the magnetic flux through the head. In this case we have to integrate $H_x$ over the head area in both the $z$ and $y$ directions with the result

$$V = \frac{\mu}{\mu + 1} - \frac{4 \pi M_s}{W} \sum_{n,m} X_n Y_m \left( \frac{k_z}{k} \right)^3 \sin \frac{k_y W h}{2} \frac{k_y W h}{2}$$

$$\times e^{-k z} \left(1 - e^{-k b}\right) \cos k_x x \cos k_y y,$$

(13)

where $v$ is the velocity of the head relative to the medium, $N$ is the number of head turns, $W$ is the width of the head, and the point $(x,y,z)$ is now at the center of the head. The same calculation can be performed for perpendicular recording with the identical result except that the factor $(k_y/k)^3$ is replaced by the factor $(k_y/k)^2$.

Equation (13) is an extremely useful equation. When combined with Eqs. (11) and (12) it allows the calculation of the usual losses (spacing, thickness, and frequency losses), but it also allows the calculation of effects due to the track width, the track separation, the location of the head relative to the track, and the effects of the transition lengths on the output voltage. The effects of a finite head gap can be included in Eq. (13) by multiplying each term by the factor, $\sin(k_y g/2)/(k_y g/2)$, where $g$ is the width of the head gap.

We will use Eq. (13) to calculate the peak output voltage in the Wallace case (one harmonic in the $x$ direction); divide the result by the Wallace voltage; and then plot the result as a function of the track separation $b$. The result is shown in Fig. 2 (dashed curve), where the other parameters are: $z=0$, $W/h=1$, $\delta A=0.01$, and $\alpha A=0.02$. It can be seen that the relative output first decreases and then levels off. These results are not unreasonable since we have shown that our result reduces to that of Wallace when $b=0$, and when $b$ becomes finite the output will decrease until there is no longer any interaction between the tracks. Figure 2 thus contains two important results: (i) It shows how far the tracks have to be separated to prevent interaction (about one wavelength); and (ii) it shows the magnitude of the loss due to the track separation (about 20% in this case).

We can now keep the separation constant ($b/h=0.5$) and see how this loss changes as the width $W$ is varied. The result is also shown in Fig. 2, and it can be seen that the loss can be considerable for small $W$ ($W<\lambda$). This is due to the fact that the Wallace voltage is linear in $W$ while our result is proportional to $W^2$ for small $W$ and constant $b$.

The results shown in Fig. 2 are just a sampling of the type of calculation that can be performed using Eq. (13). Another measurable quantity that can be calculated from the magnetic fields is the displacement of the probe tip of a magnetic force scanning tunneling microscope (MFSTM). We have shown previously how the probe tip deflections are related to the magnetic fields and how the different components of the fields can be isolated by the orientation of the probe. These calculations can now be repeated for the magnetic fields that we have calculated here. It can be shown that the different components of the fields can still be isolated for the case of a rigid triangular probe that is constrained to rotate in only one direction. These isolation effects have been confirmed experimentally. The calculations become quite complicated when the $H_y$ component is included and space precludes their presentation here. We do, however, intend to pursue these studies, both theoretically and experimentally.

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