Online Scheduling for an Energy Harvesting Link with Processing Costs

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***Abstract*—We consider scheduling for a single-user energy harvesting channel in which the transmitter incurs processing cost per unit time it is on. The presence of processing costs forces the transmitter to operate in a bursty mode. We consider *online* transmission scheduling where the transmitter knows the energy harvests only causally as they arrive, and needs to determine the optimum transmit power and the optimum burst duration on the fly. We first consider the case of independent and identically distributed (i.i.d.) Bernoulli energy arrivals, and then extend it to the case of general i.i.d. energy arrivals. We determine the *exactly optimum* online policy for Bernoulli arrivals and propose a *nearly optimum* online policy for general arrivals. The proposed policy is near-optimum in that it performs within a constant gap from the optimum policy for all energy arrivals and battery-sizes.**

I. Introduction

We consider a single-user energy harvesting channel, see Fig. 1, where the transmitter incurs a processing cost per unit time that it is on. The processing cost is the power consumed by the transmitter to be on and transmitting. This cost forces the transmitter to transmit in bursts instead of transmitting continually. The transmitter has a finite-sized battery, which is recharged by an exogenous i.i.d. energy harvesting process. In this paper, we consider the problem of *online* scheduling, where the transmitters knows the energy arrivals only causally, and needs to determine a power allocation and burst length policy with only a causal knowledge of the energy arrivals.

Offline scheduling, where the transmitter knows the energy harvesting profile non-causally ahead of time, has been considered extensively in recent literature [1]–[21], starting with the single-user channel [1]–[4], extending to multi-user and multi-hop settings [5]–[16], and incorporating processing costs [17]–[21] which lead to bursty communication as in glue-pouring in [22]. Early work in *online* scheduling, where the transmitter gets to know the energy harvesting profile only causally [3], [4], [23]–[30] has formulated the problem using dynamic programming and Markov decision process techniques.

In this paper, we follow the unique approach developed in [31]–[33] for *online* scheduling in a single-user energy harvesting system with a finite-sized battery. This approach has recently been extended to multiple access [34] and broadcast [35] settings. In this paper, we extend this approach for a single-user system with processing costs at the transmitter. Our paper may be viewed as an extension of the online setting in [32] to incorporate processing costs at the transmitter, or equivalently, as an extension of the offline setting in [17], [18] which consider processing costs to an online setting.

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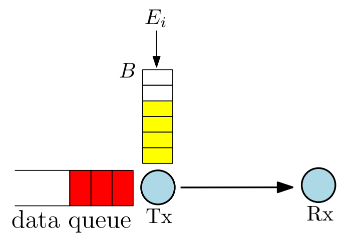

Fig. 1. System model: a single-user energy harvesting channel.

In this paper, we first consider the case of i.i.d. Bernoulli energy arrivals, where the energy arrival amount is either zero or equals the size of the battery. For this case, we determine the exactly optimal online transmission policy. We show that the optimum transmit power decreases exponentially between energy arrivals. Due to the presence of processing costs, there may exist bursts in the transmission, i.e., slots may not be fully utilized. We show that the bursty transmission can only occur in the last slot. We also show that the total transmission duration decreases as the processing cost increases.

Next, we consider the case of general i.i.d. energy arrivals, and propose a sub-optimal policy. We develop a lower bound for the performance of this sub-optimal policy, and a universal upper bound for the performance of any online policy with processing costs. We show that the developed lower and upper bounds are within a constant gap for all energy arrivals and battery sizes; hence, the proposed sub-optimal policy performs within a constant gap from the optimal online policy.

II. System Model

We consider a single-user energy harvesting channel, see Fig. 1. The transmitter has a battery of size B. Time is slotted. The amount of energy in the battery, b_i, evolves as:

$$b_{i+1} = \min\{B, b_i - \theta_i \left(P_i + \epsilon\right) + E_{i+1}\} \tag{1}$$

where E_i is the energy harvested in slot i, ϵ is the processing cost (power) per unit time, and θ_i is the duration in slot i that the transmitter is on and transmitting. In (1), $\theta_i P_i$ is the transmission energy, and $\theta_i\epsilon$ is the energy spent for being on.

The physical layer is AWGN with rate transmitted in slot i,

$$r_i = \frac{\theta_i}{2} \log\left(1 + P_i\right) \tag{2}$$

where P_i is the allocated power and θ_i is the transmission duration in slot i, which satisfy $\theta_i(P_i+\epsilon) \leq b_i$.

We first consider the case where E_i are i.i.d. Bernoulli random variables with a particular support: $\mathbb{P}[E_i = B] = p$ and $\mathbb{P}[E_i = 0] = 1-p$, that is, when energy arrives it fills the battery completely. For this case, we determine the optimal online policy in the next section. We then consider the general i.i.d. energy arrivals and propose a near-optimal policy in the following section, and prove optimality guarantees on it.

III. Optimal Strategy: Case of Bernoulli Arrivals

Due to the special i.i.d. Bernoulli energy arrival structure, when an energy arrives, it fills the battery, and resets the system. This constitutes a *renewal*. Then, from [36, Theorem 3.6.1], the long-term average rate can be found as:

$$\lim_{n\to\infty} \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} r_i\right] = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} r_i\right] \tag{3}$$

$$= p\sum_{k=1}^{\infty} p(1-p)^{k-1}\sum_{i=1}^{k} r_i \tag{4}$$

$$= \sum_{i=1}^{\infty}\sum_{k=i}^{\infty} p^2(1-p)^{k-1} r_i \tag{5}$$

$$= \sum_{i=1}^{\infty} p(1-p)^{i-1} r_i \tag{6}$$

where L is the inter-arrival time between energy harvests, which is geometric with parameter p, and $\mathbb{E}[L] = 1/p$.

Inserting (2) in (6), the online power allocation problem is:

$$\begin{aligned} \max_{\{P_i,\theta_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log\left(1+P_i\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} \theta_i(P_i+\epsilon) \leq B \\ & 0 \leq \theta_i \leq 1, \quad P_i \geq 0, \quad \forall i \end{aligned} \tag{7}$$

This optimization problem can be viewed as maximizing the *expected* transmitted rate before the next energy arrival.

The problem in (7) is a non-convex. We transform it to an equivalent convex problem by defining $\bar{P}_i = P_i\theta_i$:

$$\begin{aligned} \max_{\{\bar{P}_i,\theta_i\}} \quad & \sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{\infty} \bar{P}_i + \theta_i\epsilon \leq B \\ & 0 \leq \theta_i \leq 1, \quad \bar{P}_i \geq 0, \quad \forall i \end{aligned} \tag{8}$$

Here, $\bar{P}_i$ can be interpreted as the transmit energy allocated to the ith slot, and θ_i is the duration during which this energy is transmitted. The optimum online scheduling problem is to find the sequence of $\{\bar{P}_i, \theta_i\}_{i=1}^{\infty}$.

The Lagrangian for the problem in (8) is:

$$\mathcal{L} = -\sum_{i=1}^{\infty} p(1-p)^{i-1}\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) - \sum_{i=1}^{\infty}\gamma_i\bar{P}_i + \lambda\left(\sum_{i=1}^{\infty}\bar{P}_i + \theta_i\epsilon - B\right) - \sum_{i=1}^{\infty}\mu_i\theta_i - \sum_{i=1}^{\infty}\nu_i(1-\theta_i) \tag{9}$$

where $\lambda, \gamma_i, \mu_i, \nu_i$ are non-negative Lagrange multipliers.

First, we note that, in the optimum solution of (7), $P_i = 0$ if and only if $\theta_i = 0$. This follows because, when P_i or θ_i is zero, the objective function is zero, and choosing the other variable non-zero wastes resources. While by definition $\bar{P}_i = 0$ when either $P_i = 0$ or $\theta_i = 0$, from the preceding argument, in the optimum solution of (8), $\bar{P}_i = 0$ if and only if $P_i = 0$ and $\theta_i = 0$. Since the problem in (8) is convex, the optimal solution is found by the KKT optimality conditions. Taking the derivative of (9) with respect to $\bar{P}_i$, equating it to zero, and using the corresponding complementary slackness condition:

$$\frac{\bar{P}_i}{\theta_i} = \frac{p(1-p)^{i-1}}{\lambda} - 1 \tag{10}$$

for slots where $\theta_i > 0$. When $\theta_i = 0$, from the preceding discussion $\bar{P}_i = 0$. Noting that $P_i = \frac{\bar{P}_i}{\theta_i}$, from (10), we conclude that the optimal power is decreasing over time. Therefore, there exists a time slot when it hits zero. Hence, we define $\tilde{N}$ for which we have $\bar{P}_i, P_i, \theta_i > 0, \forall i \in \{1,\dots,\tilde{N}\}$, and $\bar{P}_i = P_i = \theta_i = 0, \forall i \in \{\tilde{N}+1,\dots\}$. Note that the transmission duration of the single-user problem with no processing costs in [32] (let us denote it as $\tilde{N}_{npc}$) forms an upper bound for the transmission duration here, i.e., $\tilde{N} \leq \tilde{N}_{npc}$. This is because, any processing costs use up energy for being on and reduce the effective battery size, and the transmission duration is an increasing function of the battery size [35].

Next, taking the derivative of (9) with respect to θ_i, we have

$$-p(1-p)^{i-1}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) + \frac{\bar{P}_i}{\theta_i}\frac{p(1-p)^{i-1}}{1+\frac{\bar{P}_i}{\theta_i}} + \lambda\epsilon - \mu_i + \nu_i = 0 \tag{11}$$

The optimal θ_i can be 0, 1, or $0<\theta_i<1$. When $0<\theta_i<1$, we have bursty transmission. In this case, from complementary slackness, we have $\mu_i = \nu_i = 0$. Then, from (10)-(11),

$$p(1-p)^{i-1}\left(\log\left(\frac{p(1-p)^{i-1}}{\lambda}\right) - 1\right) = \lambda(\epsilon - 1) \tag{12}$$

Hence, (12) should be satisfied in any slot i where $0<\theta_i<1$, i.e., where there is burstiness. Next, we note that, since the left hand side of (12) is monotonically decreasing in i, (12) can be satisfied in at most one slot. Moreover, this slot can only be the last slot. This follows from the presence of factor $p(1-p)^{i-1}$ in front of the $\log$ in (8). Hence, it is always better to fill-up (i.e., $\theta_i = 1$) earlier slots first; fractional θ_i should come later.

Next, we discuss how to solve for the optimum online policy. We just showed above that for all slots we have $\theta_i = 1$, except for possibly the last slot where $\theta_{\tilde{N}} \leq 1$. From the total energy constraint and (10), we have:

$$\sum_{i=1}^{\tilde{N}-1}\left(\frac{p(1-p)^{i-1}}{\lambda} - 1 + \epsilon\right) + \theta_{\tilde{N}}\left(\frac{p(1-p)^{\tilde{N}-1}}{\lambda} - 1 + \epsilon\right) = B \tag{13}$$

In addition, for $i \in \{1, \dots, \tilde{N}\}$, we need to satisfy:

$$p(1-p)^{i-1} \geq \lambda \quad (14)$$

$$p(1-p)^{i-1}\left(1-\log\left(\frac{p(1-p)^{i-1}}{\lambda}\right)\right)+\lambda(\epsilon-1) \leq 0 \quad (15)$$

where (14) ensures the non-negativity of power in (10), and (15) ensures the existence of non-negative Lagrange multipliers $\{\nu_i\}$ satisfying (11). Hence, we need to find the optimal $\tilde{N}, \lambda, \theta_{\tilde{N}}$ that satisfy (12), (13), (14) and (15). Towards this end, we consider the following approach: We first fix $\tilde{N}$ to be the single-user transmission duration with no processing costs in [32], i.e., $\tilde{N} = \tilde{N}_{npc}$, and solve for λ in (12) with $i = \tilde{N}$. Then, we check whether (14) and (15) are satisfied. If they are, then, we solve for $\theta_{\tilde{N}}$ from (13). If there does not exist a solution, then we reduce $\tilde{N}$ and repeat until we reach $\tilde{N} = 1$. If we do not have a solution when we reach $\tilde{N} = 1$, then this means that (12) cannot be satisfied, and we must have $\theta_{\tilde{N}} = 1$. In this case, (13) becomes:

$$\sum_{i=1}^{\tilde{N}}\left(\frac{p(1-p)^{i-1}}{\lambda}-1+\epsilon\right)=B \quad (16)$$

For this case, we solve (16) along with (14) and (15) for the largest $\tilde{N}$ and the corresponding λ.

IV. Near-Optimal Strategy: General Arrivals

Now, we consider a general i.i.d. energy arrival process E_i with recharge rate $\mathbb{E}[E_i] = \mu$. In this case, we no longer have a *renewal* structure, and finding the *exactly optimal* online policy is analytically intractable. Instead, we propose a sub-optimal online policy and prove that it performs close to optimal.

A. Sub-Optimal Policy

We first define the proposed sub-optimal online policy for Bernoulli energy arrivals and then extend it to general energy arrivals. We note from (10) that, for Bernoulli energy arrivals, the optimal total transmit power allocated decreases exponentially over time. As in [31], [32], [34], [35], this motivates us to construct a fractional power policy over time, in particular, we use a total allocated power of $Bp(1-p)^{i-1}$ in slot i. That is, we allocate a fixed p fraction of available energy in the battery to use in slot i. We then decide on the duration of the burst θ_i by solving a single-slot problem as:

$$\begin{aligned}
\max_{\bar{P}_i, \theta_i} \quad & \frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right) \\
\text{s.t.} \quad & \bar{P}_i + \theta_i\epsilon \leq Bp(1-p)^{i-1} \\
& 0 \leq \theta_i \leq 1, \quad \bar{P}_i \geq 0
\end{aligned} \quad (17)$$

In the optimal policy, the first constraint is satisfied with equality, hence $\bar{P}_i = Bp(1-p)^{i-1} - \theta_i\epsilon$, and the problem can be written only in terms of θ_i as:

$$\max_{\theta_i \in [0,1]} \quad \frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon\right) \quad (18)$$

For general energy arrivals, we allocate a fraction $q = \mu/B$ of the available energy in the battery for slot i, i.e., qb_i. Then, solve for the optimum burst θ_i in each slot as in (18):

$$\max_{\theta_i \in [0,1]} \quad \frac{\theta_i}{2}\log\left(1+\frac{qb_i}{\theta_i}-\epsilon\right) \quad (19)$$

B. A Lower Bound on the Proposed Online Policy

In Lemma 1 and Lemma 2 below, we develop multiplicative and additive lower bounds for the performance of the proposed sub-optimal algorithm for Bernoulli arrivals. In the following, we denote the solution of maximization problems in (18) and (19) for available power P as $\theta^*(P, \epsilon)$, i.e., the solution of (18) is $\theta^*(Bp(1-p)^{i-1}, \epsilon)$ and the solution of (19) is $\theta^*(qb_i, \epsilon)$.

Lemma 1 *The achievable rate with the proposed sub-optimal policy for any i.i.d. Bernoulli energy arrival process with average recharge rate of $\mu = \mathbb{E}[E_i]$ is lower bounded as,*

$$\begin{aligned}
r &\geq \frac{1}{2-\frac{\mu}{B}} \max_{\theta \in [0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) && (20) \\
&\geq \frac{1}{2} \max_{\theta \in [0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) && (21)
\end{aligned}$$

Proof: We lower bound the performance as follows. The first lower bounding step in (23) is obtained by choosing all θ_i as $\theta_i = (1-p)^{i-1}\theta^*$, where θ^* denotes $\theta^*(Bp, \epsilon)$ in short:

$$\begin{aligned}
r =& \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\max_{\theta_i \in [0,1]}\frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon\right)\right] && (22) \\
\geq& \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}\frac{\theta^*(1-p)^{i-1}}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\right] && (23) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L}(1-p)^{i-1}\right] && (24) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)p\left[\sum_{L=1}^{\infty}p(1-p)^{L-1}\sum_{i=1}^{L}(1-p)^{i-1}\right] && (25) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\left[\sum_{L=1}^{\infty}p^2(1-p)^{L-1}\frac{1-(1-p)^L}{p}\right] && (26) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\left[\sum_{L=1}^{\infty}p(1-p)^{L-1}\left(1-(1-p)^L\right)\right] && (27) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\left[\sum_{L=1}^{\infty}p\left((1-p)^{L-1}-(1-p)^{2L-1}\right)\right] && (28) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\left[p\left(\frac{1}{p}-\frac{(1-p)}{2p-p^2}\right)\right] && (29) \\
=& \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right)\left(\frac{1}{2-p}\right) && (30) \\
=& \frac{1}{2-\frac{\mu}{B}}\max_{\theta \in [0,1]}\frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) && (31)
\end{aligned}$$

which is (20). Here, (31) follows since $\mathbb{E}[E_i] = \mu = Bp$ and $\theta^* = \theta^*(Bp, \epsilon) = \theta^*(\mu, \epsilon)$. Finally, (21) follows as $\frac{\mu}{B} \geq 0$. ■

The multiplicative bound in Lemma 1 performs well when the achievable rates are small, whereas the additive bound in Lemma 2 performs well when the achievable rates are large.

Lemma 2 *The achievable rate with the proposed sub-optimal policy for any i.i.d. Bernoulli energy arrival process with average recharge rate of $\mu = \mathbb{E}[E_i]$ is lower bounded as,*

$$r \geq \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) - 0.72 - \frac{1}{2}\log^+(\epsilon) \quad (32)$$

where $\log^+(x) = \max\{\log(x), 0\}$.

Proof: We first prove for the case $\epsilon < 1$. The first lower bounding step in (34) is obtained by choosing all θ_i as $\theta_i = \theta^*$, where θ^* denotes $\theta^*(Bp, \epsilon)$ in short:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon\right)\right] \quad (33)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \frac{\theta^*}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta^*}-\epsilon\right)\right] \quad (34)$$

$$= \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log\left((1-\epsilon)\left(1+\frac{Bp(1-p)^{i-1}}{(1-\epsilon)\theta^*}\right)\right)\right] \quad (35)$$

$$= \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon)+\log\left(1+\frac{Bp(1-p)^{i-1}}{(1-\epsilon)\theta^*}\right)\right] \quad (36)$$

$$\geq \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon)+\log\left(1+\frac{Bp}{(1-\epsilon)\theta^*}\right) + \log\left((1-p)^{i-1}\right)\right] \quad (37)$$

$$\geq \frac{\theta^*}{2}\frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \log(1-\epsilon)+\log\left(1+\frac{Bp}{(1-\epsilon)\theta^*}\right)\right] - 0.72 \quad (38)$$

$$= \frac{\theta^*}{2}\log\left(1+\frac{Bp}{\theta^*}-\epsilon\right) - 0.72 \quad (39)$$

which is (32), since $\mathbb{E}[E_i] = \mu = Bp$, $\theta^* = \theta^*(Bp, \epsilon)$, and $\log^+(\epsilon) = 0$ in this case.

Next, we prove for the case $\epsilon \geq 1$:

$$r = \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon\right)\right] \quad (40)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{\frac{Bp(1-p)^{i-1}}{\theta_i}-\epsilon}{\epsilon}\right)\right] \quad (41)$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{Bp(1-p)^{i-1}}{\theta_i\epsilon}-1\right)\right] \quad (42)$$

$$\geq \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(\frac{Bp}{\theta_i}\right)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log(\epsilon)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(\frac{1}{(1-p)^{i-1}}\right)\right] \quad (43)$$

$$= \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(\frac{Bp}{\theta_i}\right)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \frac{1}{2}\log(\epsilon)\right] - \frac{1}{\mathbb{E}[L]}\mathbb{E}\left[\sum_{i=1}^{L} \frac{1}{2}\log\left((1-p)^{i-1}\right)\right] \quad (44)$$

$$= \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(\frac{Bp}{\theta}\right) - \frac{1}{2}\log(\epsilon) - 0.72 \quad (45)$$

$$\geq \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{Bp}{\theta}-\epsilon\right) - \frac{1}{2}\log(\epsilon) - 0.72 \quad (46)$$

which is (32), since $\log^+(\epsilon) = \log(\epsilon)$ in this case. Here, (41) follows since at the maximum $\frac{Bp(1-p)^{i-1}}{\theta_i} - \epsilon$ is non-negative and $\epsilon \geq 1$, (43) follows since for any three positive functions $a(x), b(x), c(x)$, we have: $\max_x[a(x) - b(x) - c(x)] \geq \max_x a(x) - \max_x b(x) - \max_x c(x)$, and (46) follows since we added a negative term $(1-\epsilon)$ inside the log. ■

We next show that i.i.d. Bernoulli energy arrivals yield the lowest rate over all i.i.d. energy arrivals with the same mean. The proof follows by the approach in [32, Proposition 4] as,

$$f(x) = \max_{\theta_i\in[0,1]} \frac{\theta_i}{2}\log\left(1+\frac{qx}{\theta_i}-\epsilon\right) \quad (47)$$

is concave in x. The concavity of $f(x)$ follows since it is equivalent to the maximization of $\frac{\theta_i}{2}\log\left(1+\frac{\bar{P}_i}{\theta_i}\right)$ over the feasible set $\bar{P}_i + \theta_i\epsilon \leq qx$, $0 \leq \theta_i \leq 1$, $\bar{P}_i \geq 0$. The objective of this equivalent problem is jointly concave in $\theta_i, \bar{P}_i$, and the constraint set is affine in x, θ_i and $\bar{P}_i$. Then, it follows that $f(x)$ is concave in x; see also [37, Section 3.2.5].

Lemma 3 *The rate of the proposed sub-optimal policy with any i.i.d. energy arrival process is no smaller than that with an i.i.d. Bernoulli energy arrival process of the same mean.*

Combining Lemmas 1, 2, and 3, we have the following general theorem for arbitrary i.i.d. energy arrival processes.

Theorem 1 *The achievable rate with the proposed sub-optimal policy for any arbitrary i.i.d. energy arrival process with average recharge rate of $\mu = \mathbb{E}[E_i]$ is lower bounded as in (20) and (32).*

C. An Upper Bound for Online Policies

In Theorem 2 below, we develop a universal upper bound for the performance of any online policy in terms of $\mathbb{E}[E_i] = \mu$.

Theorem 2 *For a recharge rate of $\mathbb{E}[E_i] = \mu$, the achievable rate of any online algorithm is upper bounded as,*

$$r \leq \max_{\theta\in[0,1]} \frac{\theta}{2}\log\left(1+\frac{\mu}{\theta}-\epsilon\right) \quad (48)$$

Proof: We consider the rate of the optimum offline algorithm which upper bounds the rates achievable by any online algorithm. We consider the following larger than actual feasible

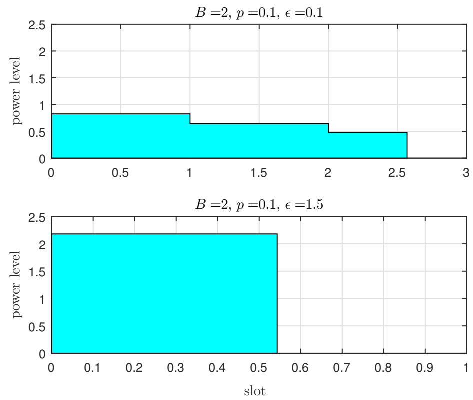

Fig. 2. Optimum online power allocation for i.i.d. Bernoulli arrivals.

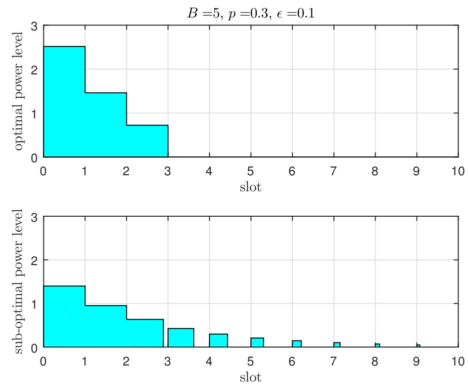

Fig. 3. Optimum online power allocation versus sub-optimal power allocation for i.i.d. Bernoulli arrivals.

region for the offline policy by neglecting the no-energy-overflow constraints due to the finite-sized battery [2], [3]:

$$\mathcal{F}^n \triangleq \left\{ \{\bar{P}_i, \theta_i\}_{i=1}^n : \frac{1}{m}\sum_{i=1}^{m} \bar{P}_i + \theta_i\epsilon \leq \frac{1}{m}\sum_{i=1}^{m} E_i,\ \forall m \right\} \quad (49)$$

Then, we consider the further larger feasible set by keeping only the bound for $m = n$, and starting with a full battery B,

$$\mathcal{G}^n \triangleq \left\{ \{\bar{P}_i, \theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^{n} \bar{P}_i + \theta_i\epsilon \leq \frac{1}{n}\left(\sum_{i=1}^{n} E_i + B\right) \right\} \quad (50)$$

Then, we have:

$$r \leq \lim_{n\to\infty} \max_{\{\bar{P}_i,\theta_i\}_{i=1}^n \in \mathcal{G}^n} \frac{1}{n}\sum_{i=1}^{n} \frac{\theta_i}{2}\log\left(1 + \frac{\bar{P}_i}{\theta_i}\right) \quad (51)$$

Since the energies E_i are i.i.d., from strong law of large numbers, for all $\delta > 0$, there exists an integer N such that, for all $n \geq N$, we have $\frac{1}{n}\left(\sum_{i=1}^{n} E_i + B\right) \leq \mu + \delta$. Hence, for large enough n, i.e., $n \geq N$, we have $\mathcal{G}^n$ to be

$$\mathcal{G}^n \triangleq \left\{ \{\bar{P}_i, \theta_i\}_{i=1}^n : \frac{1}{n}\sum_{i=1}^{n} \bar{P}_i + \theta_i\epsilon \leq \mu + \delta \right\} \quad (52)$$

Then, from the joint concavity of the objective function, it is maximized when all $\theta_i = \theta$ and all $\bar{P}_i = \bar{P}$. Hence, we have $\bar{P} + \theta\epsilon \leq \mu + \delta$. Since this is valid for all $\delta > 0$, we take its limit to zero, which gives the desired result in (48). ■

Finally, the additive lower bound in Theorem 1 (i.e., (32)) together with the general upper bound in Theorem 2 (i.e., (48)) imply that there is a constant gap between the bounds, hence the sub-optimal policy is within a constant gap of the optimal.

V. Numerical Results

In this section, we illustrate our results using several numerical examples. We first show the optimal policy for Bernoulli energy arrivals. We fix the battery size to $B = 2$ and the probability of energy arrival to $p = 0.1$. We show the optimal policy in Fig. 2 for ϵ values of 0.1 and 1.5. As the processing cost increases, the transmission time decreases. When $\epsilon = 0.1$, the optimal power is decreasing and is non-zero for a total duration of 2.6 slots. However, when the processing cost is 1.5, the transmission duration decreases to 0.55 slots.

Next, in Fig. 3, for the case of Bernoulli energy arrivals, we show the optimal policy versus the proposed sub-optimal policy. Here, we have $B = 3$, $p = 0.3$, $\epsilon = 0.1$. In the sub-optimal policy the energy is spread over more (infinite) slots.

In Figs. 4 and 5, we show the performance of the proposed sub-optimal policy and the optimal policy in terms of the expected rate versus the battery size. We fix $p = 0.1$ and show the performance for processing costs of $\epsilon = 1$ and $\epsilon = 10$ in Figs. 4 and 5. We note that, for the case of Bernoulli arrivals, the performance of the proposed sub-optimal policy is quite close to the performance of the optimal policy, in fact, much closer than the derived theoretical bounds show.

In Figs. 4 and 5, we further plot two other sub-optimal schemes. The first scheme uses the same total fractional power as our proposed policy but fixes $\theta_i = 1$ for all i (i.e., neglects the processing cost effect) and transmits whenever it is feasible to transmit. The second scheme also uses the same total fractional power as our proposed policy but uses a fractional decreasing burstiness as $\theta_i = (1-p)^{i-1}\theta^*$. We observe that both of these policies perform worse than our proposed policy. We observe that the policy with $\theta = 1$ performs close to the optimal when the value of processing cost is negligible with respect to the battery size, i.e., for large battery sizes. However, for small battery sizes, e.g., B in $[1, 10]$ when $\epsilon = 1$ and B in $[1, 100]$ when $\epsilon = 10$, this algorithm performs poorly.

In Figs. 4 and 5, we also plot the performance of the proposed sub-optimal policy when the energy arrivals come from a continuous uniform distribution (non-Bernoulli) with the same mean as the Bernoulli energy arrivals. As expected, the rate is higher for the case of general energy arrivals compared to Bernoulli energy arrivals with the same mean.

Finally, we show the performance of our scheme versus the processing cost in Fig. 6. The gap between the optimal and the sub-optimal decreases for high processing costs.

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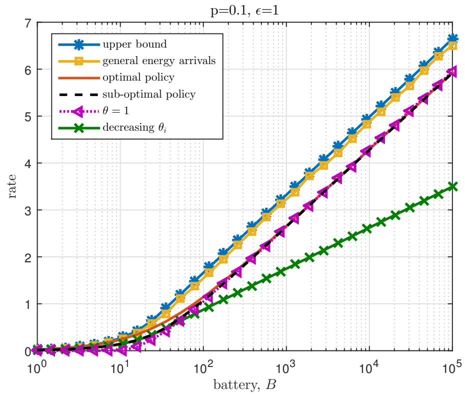

Fig. 4. Optimum online policy versus proposed sub-optimum online policy.

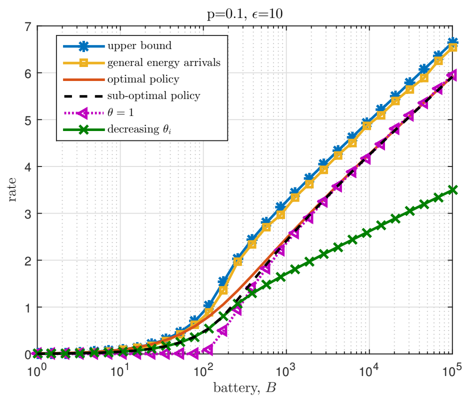

Fig. 5. Optimum online policy versus proposed sub-optimum online policy.

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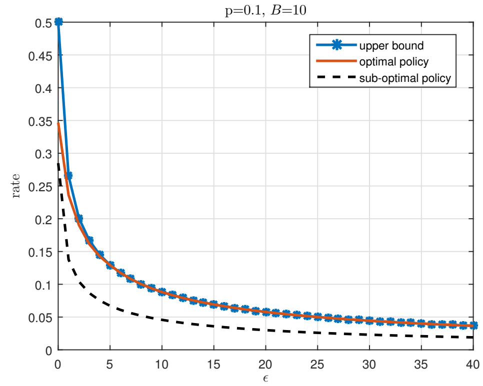

Fig. 6. Performance versus processing cost for i.i.d. Bernoulli arrivals.

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