Explicit and Implicit Temperature Constraints in Energy Harvesting Communications

Abdulrahman Baknina[1], Omur Ozel[2], and Sennur Ulukus[1]

[1]Department of Electrical and Computer Engineering, University of Maryland, College Park, MD
[2]Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA

***Abstract*—We consider an energy harvesting communication system where the temperature dynamics is governed by the transmission power policy. Different from the previous work, we consider a discrete time system where transmission power is kept constant in each slot. We consider two models that capture different effects of temperature. In the first model, the temperature is constrained to be below a critical temperature at all time instants; we coin this model as *explicit temperature constraint model*. We investigate throughput optimal power allocation for multiple energy arrivals under general, as well as temperature and energy limited regimes. In the second model, we consider the effect of the temperature on the channel quality; we coin this model as *implicit temperature constraint model*. As the dynamic range of the temperature changes significantly, the change in the thermal noise becomes non-negligible, affecting the signal to interference plus noise ratio (SINR). In effect, transmitted signals contribute as interference for all subsequent slots. In this case, we investigate throughput optimal power allocation under general, as well as low and high SINR regimes.**

I. Introduction

We study two different effects of temperature change on the optimal power allocation, and hence, on the system performance of a single-user energy harvesting system. We consider explicit and implicit temperature constrained systems. In the explicit temperature constrained model, a peak temperature constraint prevents the system from overheating. In the implicit temperature constrained model, the effect of the temperature on the channel quality controls the system temperature.

The scheduling-theoretic approach for energy harvesting communications was studied in various settings, see [1]–[15]. Previous works considered single-user channel [1]–[4], broadcast channel [5], multiple access channel [6], [7], interference channel [8], two-hop channel [9], [10], two-way channel [11], [12], and diamond channel [13]. Temperature effects are studied in [14], [15]. In [14], a peak temperature constraint is considered and the optimal continuous power allocation is studied. Although extensive insights and properties of the optimal policy were derived, only the single energy arrival case was fully solved. The fact that the power can take any continuous function makes the problem challenging, as the problem then becomes a functional optimization problem and the optimal such function should be obtained. In this paper, we consider the slotted version of the temperature evolution model considered in [14], [15].

This work was supported by NSF Grants CNS 13-14733, CCF 14-22111, CCF 14-22129, and CNS 15-26608.

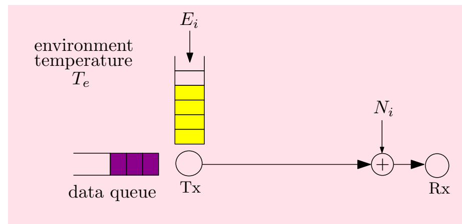

Fig. 1. System model: the system heats up due to data transmission.

In the first model we consider here, which we coin as the *explicit temperature constrained model*, we consider an explicit peak temperature constraint. Increasing the transmission power increases the throughput and the temperature. Higher temperature levels mean smaller admissible transmission power levels for future slots. This model is the slotted version of [14]. We study the optimal power allocation for the multiple energy arrival case, for which we develop a generalized water-filling algorithm. Then, we study sufficient conditions under which the system operates in the limiting cases of non-binding temperature or energy constraints. When the temperature constraint is not binding, the problem reduces to the single-user energy harvesting channel studied in [1]. When the energy constraint is not binding, the system effectively becomes a temperature constrained system with all the needed energy arriving before the communication begins. In this case, we show that the optimal powers are bounded and non-increasing, and the temperature of the system is non-decreasing.

In the second model we consider here, which we coin as the *implicit temperature constrained model*, the temperature is not explicitly constrained, however, the temperature affects the thermal noise which affects the channel quality. This arises when the dynamic range of the temperature is large, and is similar to that presented in [16] but in a scheduling-theoretic setting. In this case, the transmit powers used in previous channel uses affect the thermal noise and therefore the channel quality in future channel uses, and hence, the channel becomes a *use dependent* or *action dependent* channel, see [17]–[19].

Specifically, each slot sees the previous transmissions as interference through a certain temperature filter. This filter arises naturally from discretizing time into time slots. For the general signal to interference plus noise ratio (SINR), the problem is non-convex and is a signomial problem for which we achieve a local optimal solution using the single condensation method proposed in [20]. We then propose a heuristic algorithm which improves upon the local optimal solution and may achieve the global optimal solution. Then, we study the extreme settings of low and high SINR regimes. We show that for low SINR regime it is optimal to transmit zero powers for all the slots except for the last, in which all the harvested energy is transmitted. For the high SINR regime, the optimal solution can be found using geometric programming.

II. System Model

We consider an energy harvesting communication system in which the transmitter harvests energy $\tilde{E}_i$ in the ith slot, see Fig. 1. We consider the same temperature model considered at [14], [15]. In this model, the temperature, $T(t)$, is defined by the following differential equation,

$$\frac{dT(t)}{dt} = ap(t) - b(T(t) - T_e) \tag{1}$$

where T_e is the environment temperature, $T(t)$ is the temperature at time t, and a, b are non-negative constants. With the initial temperature $T(0) = T_e$, the solution of (1) is:

$$T(t) = e^{-bt} \int_0^t e^{b\tau} ap(\tau) d\tau + T_e \tag{2}$$

In what follows we assume that the duration of each slot is equal to Δ, which can take any positive value. Let us define $T_i \triangleq T(i\Delta)$ as the temperature level by the end of the ith slot, $P_i \triangleq P(i\Delta)$ as the power level used in the ith slot. Using (2), T_i can be expressed as:

$$T_i = \alpha T_{i-1} + \beta P_i + \gamma \tag{3}$$

where $\alpha = e^{-b\Delta}$, $\beta = \frac{a}{b}[1-\alpha]$ and $\gamma = T_e[1-\alpha]$.

The effect of Δ in (3) appears through the constants α, β, γ. As the slot duration increases, the values of β, γ increase while the value of α decreases; as the slot duration increases, the temperature at the end of the slot becomes more dependent on the power transmitted within this slot and less dependent on the initial temperature at the beginning of the slot.

We now eliminate the previous temperature readings in T_i making the temperature a function of the powers only. We can do this by recursively substituting by T_{i-1} in T_i to have

$$T_k = \beta \sum_{i=1}^{k} \alpha^{k-i} P_i + T_e \tag{4}$$

This formula shows that the temperature at the end of each slot depends on the power transmitted in this slot and all previous slots through an exponentially decaying *temperature filter*. We note that this is the same formula that was developed in [16] in which the slot duration was assumed to be unity.

III. Explicit Peak Temperature Constraint

We now consider the model in which we have an energy harvesting transmitter with a peak temperature constraint. The noise variance is the same throughout the communication session and is set to σ^2. We consider a slotted system with a constant power per slot. It follows from [14, equation (47)], that the temperature is monotone within the slot duration. Hence, for the peak temperature constrained case, it suffices to constrain the temperature only at the end of each slot; we begin the communication with the system having temperature T_e. In this case, the problem can be written as,

$$\begin{aligned} \max_{\{P_i\} \geq 0} \quad & \sum_{i=1}^{D} \frac{\Delta}{2} \log\left(1 + \frac{P_i}{\sigma^2}\right) \\ \text{s.t.} \quad & T_k \leq T_c,\ \sum_{i=1}^{k} \Delta P_i \leq \sum_{i=1}^{k} \tilde{E}_i,\ \forall k \end{aligned} \tag{5}$$

where Δ in the objective function and the energy constraint is the slot duration. In what follows, without loss of generality, we drop Δ since it is just a constant multiplied in the objective function and we define $E_i = \frac{\tilde{E}_i}{\Delta}$.

We rewrite problem (5) making use of (4) as,

$$\begin{aligned} \max_{\{P_i\} \geq 0} \quad & \sum_{i=1}^{D} \frac{1}{2} \log\left(1 + \frac{P_i}{\sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} \alpha^{k-i} P_i \leq \frac{T_c - T_e}{\beta},\ \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} E_i,\ \forall k \end{aligned} \tag{6}$$

In the last slot, either the temperature or the energy constraint has to be satisfied with equality. Otherwise, we can increase one of the powers until one of the constraints is met with equality and this strictly increases the objective function.

This problem is a convex problem in the powers, which can be solved optimally using the KKTs and the Lagrangian is:

$$\begin{aligned} \mathcal{L} = & -\sum_{i=1}^{D} \log\left(1 + \frac{P_i}{\sigma^2}\right) + \sum_{k=1}^{D} \lambda_k \left(\sum_{i=1}^{k} \alpha^{k-i} P_i - \frac{T_c - T_e}{\beta}\right) \\ & + \sum_{k=1}^{D} \mu_k \left(\sum_{i=1}^{k} P_i - \sum_{i=1}^{k} E_i\right) \end{aligned} \tag{7}$$

Differentiating with respect to P_i and equating to zero we get,

$$P_i = \frac{1}{\alpha^{-i} \sum_{k=i}^{D} \lambda_k \alpha^k + \sum_{k=i}^{D} \mu_k} - \sigma^2 \tag{8}$$

In the optimal solution, if neither constraint was tight in slot $i < D$, then the power in slot $i+1$ is strictly less than the power in slot i. This follows from complementary slackness since if at slot i, if both constraints were not tight then we have $\lambda_i = \mu_i = 0$ which, using (8), implies that $P_i > P_{i+1}$.

To find the optimal solution, we begin with an initial feasible power allocation. If for this power allocation there exist non-negative Lagrange multipliers which satisfy (8) and the complementary slackness conditions, then this is the optimal power allocation. Otherwise, these power allocations should be modified by pouring water away from the slots with negative Lagrange multipliers to the slots with higher Lagrange multipliers until non-negative Lagrange multipliers are found and the complementary slackness conditions are satisfied. Since this problem is a convex optimization problem, any solution for the KKTs achieve the global maximum.

A. Energy Limited Case

In this subsection, we study a sufficient condition under which the system becomes energy limited. For all slots j in which the following is satisfied

$$\sum_{i=1}^{j} E_i \leq \frac{T_c - T_e}{\beta} \tag{9}$$

the temperature constraint cannot be tight. In particular, when it is satisfied for $k = D$, then the temperature constraint can be completely removed from the system. To prove this, we assume for the sake of contradiction that we have at slot j $\sum_{i=1}^{j} E_i \leq \frac{T_c - T_e}{\beta}$ while the temperature constraint is tight, which implies:

$$\frac{T_c - T_e}{\beta} = \sum_{i=1}^{j} \alpha^{j-i} P_i < \sum_{i=1}^{j} P_i \leq \sum_{i=1}^{j} E_i \tag{10}$$

which contradicts the assumption $\sum_{i=1}^{j} E_i \leq \frac{T_c - T_e}{\beta}$. The strict inequality follows since $\alpha < 1$. The structure of the optimal solution for this case is studied in [1].

B. Temperature Limited Case

In this part, we first study a sufficient condition for problem (6) to be temperature limited. The energy constraint is never tight if the following condition is satisfied:

$$\frac{T_c - T_e}{\beta} < \frac{\sum_{i=1}^{k} E_i}{k},\ \forall k \in \{1, \dots, D\} \tag{11}$$

For the temperature limited case, an upper bound on the transmission powers is equal to $\frac{T_c - T_e}{\beta}$. Hence, (11) is sufficient to satisfy $\sum_{i=1}^{k} P_i < \sum_{i=1}^{k} E_i$.

In what follows, we study the structure of the optimal policy for the temperature limited case. In the last slot, the temperature constraint is satisfied with equality. The optimal powers are monotonically decreasing in time. The proof follows by contradiction. Assume for some index j that we have $P_j^* < P_{j+1}^*$. We now form another policy, denoted as $\{\bar{P}_i\}$, which has $\bar{P}_i = P_i^*$ for all slots $i \neq j, j+1$, while we change the powers of slots $j, j+1$ to be $\bar{P}_j = P_j^* + \delta$ and $\bar{P}_{j+1} = P_{j+1}^* - \delta$ for small enough δ. This δ always exists as $P_j^* < P_{j+1}^*$ implies that $\sum_{k=1}^{j} \alpha^{j-k} P_k^* < \frac{T_c - T_e}{\beta}$. Since the objective function is strictly concave, this new policy yields a strictly higher objective function, which contradicts the optimality of $P_j^* < P_{j+1}^*$. Now it remains to check that with this new policy, the temperature constraint is still feasible for any slot $k \geq j+1$ which follows from:

$$\sum_{i=1, \neq j, j+1}^{k} \alpha^{k-i} \bar{P}_i + \alpha^{k-j} \bar{P}_j + \alpha^{k-j-1} \bar{P}_{j+1}$$

$$< \sum_{i=1, \neq j, j+1}^{k} \alpha^{k-i} P_i^* + \alpha^{k-j} P_j^* + \alpha^{k-j-1} P_{j+1}^* \tag{12}$$

$$= \sum_{i=1}^{k} \alpha^{k-i} P_i^* < \frac{T_c - T_e}{\beta} \tag{13}$$

Moreover, the optimal temperature levels are non-decreasing in time. To prove this, it suffices to show that:

$$\sum_{i=1}^{k} \alpha^{k-i} P_i^* \leq \sum_{i=1}^{k+1} \alpha^{k+1-i} P_i^*, \quad \forall k = \{1, \dots, D-1\} \tag{14}$$

We rewrite (14) as follows,

$$(1-\alpha) \sum_{i=1}^{k} \alpha^{k-i} P_i^* \leq P_{k+1}^*, \quad \forall k = \{1, \dots, D-1\} \tag{15}$$

Since, we know that the last slot has to be satisfied with equality then we know $\sum_{i=1}^{D} \alpha^{D-i} P_i^* = \frac{T_c - T_e}{\beta}$. Hence, for the constraint at $k = D-1$ we have:

$$\sum_{i=1}^{D-1} \alpha^{D-1-i} P_i^* \leq \frac{T_c - T_e}{\beta} = \sum_{i=1}^{D} \alpha^{D-i} P_i^* \tag{16}$$

which can be written as follows,

$$(1-\alpha) \sum_{i=1}^{D-1} \alpha^{D-1-i} P_i^* \leq P_D^* \tag{17}$$

which proves (15) for $k = D-1$. Now assume for the sake of contradiction that (15) is false for $k = D-2$, i.e.:

$$P_{D-1}^* < (1-\alpha) \sum_{i=1}^{D-2} \alpha^{D-2-i} P_i^* \tag{18}$$

Substituting this in (17), we get:

$$P_{D-1}^* = \alpha P_{D-1}^* + (1-\alpha) P_{D-1}^* \tag{19}$$

$$< \alpha(1-\alpha) \sum_{i=1}^{D-2} \alpha^{D-2-i} P_i^* + (1-\alpha) P_{D-1}^* \tag{20}$$

$$= (1-\alpha) \sum_{i=1}^{D-1} \alpha^{D-1-i} P_i^* \leq P_D^* \tag{21}$$

But since we know that in the optimal policy the powers are non-decreasing, this is a contradiction and (15) holds for $k = D-2$. The same argument follows for any $k < D-2$.

In optimal solution, if the constraint is satisfied with equality for two consecutive slots then the power in the second slot must be equal to $(1-\alpha)\frac{T_c - T_e}{\beta}$. To obtain this, the two consecutive constraints which are satisfied with equality are solved simultaneously for the power in the second slot. Hence, when the temperature hits the critical temperature, the optimal transmission power in all the subsequent slots becomes constant and equal to $(1-\alpha)\frac{T_c - T_e}{\beta}$. This follows since the temperature is increasing, thus whenever the constraint becomes tight, it remains tight for all subsequent slots. We now conclude that the transmission power is bounded as

$$(1-\alpha)\frac{T_c - T_e}{\beta} \leq P_i \leq \frac{T_c - T_e}{\beta}, \quad \forall i = \{1, \dots, D\} \tag{22}$$

The lower bound follows from the discussion above while the upper bound follows from the feasibility of the constraints.

We then proceed to find the optimal power allocations. Since the problem is convex, a necessary and sufficient condition is to find a solution satisfying the KKTs. The optimal power is given by setting $\{\mu_i\} = 0$ in (8).

To this end we present an approach to obtain the optimal powers. We use line search to search over the time slot at which the temperature constraint becomes tight, which we denote as i^*. Then slots $i = \{i^* + 1, \ldots, D\}$ have power allocation equal to $(1-\alpha)\frac{T_c - T_e}{\beta}$, while the power allocations for slots $i = \{1, \ldots, i^*\}$ are strictly decreasing and strictly higher than $(1-\alpha)\frac{T_c - T_e}{\beta}$. Hence, we initialize $i^* = D$ and search for a solution for the powers satisfying the KKTs. If we obtain a solution then we stop and this is the optimal solution. Otherwise, we decrease i^* by one and repeat the search.

IV. Implicit Temperature Constraint

We now consider the case when the dynamic range of the temperature increases. In this case, we need to consider the change in the thermal noise of the system due to temperature changes. The thermal noise is linearly proportional to the temperature [21, Chapter 11]. The objective function is:

$$\sum_{i=1}^{D} \frac{1}{2} \log \left(1 + \frac{P_i}{cT_{i-1} + \sigma^2}\right) \tag{23}$$

where c is the proportionality constant between the thermal noise and the temperature. In this setting, the noise variance in each slot is determined by the value of the temperature at the beginning of the slot. The maximum temperature the system can reach is equal to $T_{max} \triangleq \beta \sum_{i=1}^{D} E_i + T_e$. This occurs when the transmitter transmits all its energy arrivals in the last slot. The value of T_{max} is useful in determining the maximum possible temperature on the system. As we show in the low SINR case later, the optimal power allocation results in system temperature equal to T_{max}.

Using (4) in (23), the problem can now be written in terms of only transmission powers as follows:

$$\begin{aligned} \max_{\{P_i\}} \quad & \sum_{i=1}^{D} \frac{1}{2} \log \left(1 + \frac{P_i}{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + T_e\right) + \sigma^2}\right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} E_i, \quad P_k \geq 0, \ \forall k \in \{1, \ldots, D\} \end{aligned} \tag{24}$$

The problem in this form highlights the effect of previous transmissions on subsequent slots. The transmission power at time i appears as an *interfering term* at slot indices greater than i with an exponentially decaying weight due to the *filtering* in the temperature. This problem is non-convex and determining the global optimal solution is generally a difficult task. Next, we adapt the signomial programming based iterative algorithm in [20] for the energy harvesting case. This algorithm provably converges to a local optimum point.

The problem in (24) can be written in the following equivalent signomial minimization problem

$$\begin{aligned} \min_{\{P_i\}} \quad & \prod_{i=1}^{D} \left(\frac{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + T_e\right) + \sigma^2}{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + T_e\right) + \sigma^2 + P_i} \right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} E_i, \quad P_k \geq 0, \ \forall k \in \{1, \ldots, D\} \end{aligned} \tag{25}$$

The objective function in (25) is a signomial function which is a ratio between two posynomials. Note also that the energy harvesting constraints in (25) are posynomials in p_i.

In each iteration we approximate the objective by a posynomial. We do this by approximating the posynomial in the denominator by a monomial. Appropriate choice of an approximation which satisfies the conditions in [22] guarantees convergence to a local optimal solution. Let us denote the posynomial in the ith denominator evaluated using a power vector $\mathbf{P}$ by $u_i(\mathbf{P})$, i.e., we have

$$u_i(\mathbf{P}) \triangleq \sum_{k=1}^{i+1} v_k^i(\mathbf{P}) = c\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + P_i + cT_e + \sigma^2 \tag{26}$$

where for $k = \{1, \ldots, i-1\}$ we have $v_k^i(\mathbf{P}) = c\beta\alpha^{i-1-k} P_k$, $v_i^i(\mathbf{P}) = P_i$ and $v_{i+1}^i(\mathbf{P}) = cT_e + \sigma^2$.

Using the arithmetic-geometric mean inequality we approximate each posynomial by a monomial as follows:

$$u_i(\mathbf{P}) \geq \left(\prod_{k=1}^{i-1} \left(\frac{c\beta\alpha^{i-1-k} P_k}{\theta_k^i} \right)^{\theta_k^i} \right) \left(\frac{P_i}{\theta_i^i} \right)^{\theta_i^i} \left(\frac{cT_e + \sigma^2}{\theta_{i+1}^i} \right)^{\theta_{i+1}^i} \tag{27}$$

where $\sum_{k=1}^{i+1} \theta_k^i = 1$ for all $i = \{1, \ldots, D\}$.

We now solve the previous problem iteratively where we initialize the power allocation to any feasible power allocation $\mathbf{P}^0$. Then, we approximate the posynomials $u_i(\mathbf{P}^0)$ using the arithmetic-geometric mean inequality shown above. In each iteration j where the power allocation is $\mathbf{P}^j$ we choose θ_k^i as a function of the posynomials and power allocation as follows:

$$\theta_k^i(\mathbf{P}^j) = \frac{v_k^i(\mathbf{P}^j)}{u_i(\mathbf{P}^j)} \tag{28}$$

which satisfies $\sum_{k=1}^{i+1} \theta_k^i(\mathbf{P}^j) = 1$. This choice of $\theta_k^i(\mathbf{P}^j)$ guarantees that the iterations converge to a KKT point of the original problem [22]. In particular, for each iteration this is a geometric program and as required by [22], this can be transformed into a convex problem; see also [20].

The above iterative approach converges to a local optimal solution. Achieving the global optimal solution is of exponential complexity. Alternatively, to get to the optimal solution, an approach introduced in [23] can be used. This approach solves the following problem iteratively:

$$\begin{aligned} \min_{\{P_i\}, t} \quad & t \\ \text{s.t.} \quad & O(\mathbf{P}) \leq t, \ t \leq \frac{t_0}{\alpha}, \ \sum_{i=1}^{k} P_i \leq \sum_{i=1}^{k} E_i, \quad P_k \geq 0 \end{aligned} \tag{29}$$

where $O(\mathbf{P})$ is the objective function of (25) and α is chosen to be a number which is slightly more than 1 and t_0 can be initialized to be the solution of problem (25) and then updated as the optimal solutions resulting from (29).

A. Low SINR Case

The low SINR case occurs when the incoming energies are very small with respect to the noise variance. In this case, an approximation to the logarithm function in the objective function is the linear function. Hence, the objective function can be rewritten as, which is similar to [24, equation (14)]:

$$\sum_{i=1}^{D} \frac{P_i}{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + T_e\right) + \sigma^2} \tag{30}$$

In this case, the optimal allocation is zero for all time slots, except in slot D in which it is equal to $P_D = \sum_{i=1}^{D} E_i$. This follows since (30) can be upper bounded by

$$\sum_{i=1}^{D} \frac{P_i}{cT_e + \sigma^2} \leq \frac{\sum_{i=1}^{D} E_i}{cT_e + \sigma^2} \tag{31}$$

and it is achieved by the specified allocation. Hence, a sufficient condition to have a low SINR regime is $\sum_{i=1}^{D} E_i \ll cT_e + \sigma^2$. Note also that the non-causal energy arrival knowledge is *not necessary* here as all energy arrivals are used only at the very last slot. The temperature at the end of the communication session is equal to $T_{max} = \beta \sum_{i=1}^{D} E_i + T_e$.

B. High SINR Case

When the values of c and σ are small, SINR is high and we approximate the objective function by ignoring 1 inside the logarithm. In this case, the problem in (24) has the Lagrangian:

$$\begin{aligned}\mathcal{L} = &-\sum_{i=1}^{D} \frac{1}{2} \log \left(\frac{P_i}{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} P_k + T_e\right) + \sigma^2} \right) \\ &+ \sum_{k=1}^{D} \mu_k \left(\sum_{i=1}^{k} P_i - \sum_{i=1}^{k} E_i \right)\end{aligned} \tag{32}$$

Taking the derivative with respect to P_i gives,

$$\frac{\partial \mathcal{L}}{\partial P_i} = -\frac{1}{P_i} + \sum_{j=i+1}^{D} \frac{c\beta\alpha^{j-1-i}}{c\left(\beta \sum_{k=1}^{j-1} \alpha^{j-1-k} P_k + T_e\right) + \sigma^2} + \sum_{k=i}^{D} \mu_k \tag{33}$$

and then equating to zero gives:

$$\frac{1}{P_i} - \sum_{j=i+1}^{D} \frac{c\beta\alpha^{j-1-i}}{c\left(\beta \sum_{k=1}^{j-1} \alpha^{j-1-k} P_k + T_e\right) + \sigma^2} = \sum_{k=i}^{D} \mu_k \tag{34}$$

Although this problem is non-convex, it is a geometric program and we show next that any local optimal solution for this problem is also a global optimal solution. To show this we propose the following equivalent problem:

$$\begin{aligned}\min_{\{x_i\}} \quad & \sum_{i=1}^{D} \frac{1}{2} \log \left(\frac{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} e^{x_k} + T_e\right) + \sigma^2}{e^{x_i}} \right) \\ \text{s.t.} \quad & \sum_{i=1}^{k} e^{x_i} \leq \sum_{i=1}^{k} E_i, \ x_i \in \mathbb{R} \ \ \forall k \in \{1, \ldots, D\}\end{aligned} \tag{35}$$

This equivalent problem is obtained by substituting $P_i = e^{x_i}$. Problem in (35) is a convex optimization problem since the objective is a convex function in the form of a log-sum-exponent and the constraint set is a convex set [25]. Hence, the KKTs are necessary and sufficient for global optimality. We next show that any solution for the KKTs of the original non-convex problem also yields a global optimal solution. To prove this, we first write the Lagrangian of problem (35) as:

$$\begin{aligned}\mathcal{L} = &\sum_{i=1}^{D} \frac{1}{2} \log \left(\frac{c\left(\beta \sum_{k=1}^{i-1} \alpha^{i-1-k} e^{x_k} + T_e\right) + \sigma^2}{e^{x_i}} \right) \\ &+ \sum_{k=1}^{D} \nu_k \sum_{i=1}^{k} \left(e^{x_i} - E_i \right)\end{aligned} \tag{36}$$

Taking the derivative with respect to x_i gives,

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_i} = &-1 + \sum_{j=i+1}^{D} \frac{c\beta\alpha^{j-1-i} e^{x_i}}{c\left(\beta \sum_{k=1}^{j-1} \alpha^{j-1-k} e^{x_k} + T_e\right) + \sigma^2} \\ &+ e^{x_i} \sum_{k=i}^{D} \nu_k\end{aligned} \tag{37}$$

which provides the following necessary condition:

$$-e^{-x_i} + \sum_{j=i+1}^{D} \frac{c\beta\alpha^{j-1-i}}{c\left(\beta \sum_{k=1}^{j-1} \alpha^{j-1-k} e^{x_k} + T_e\right) + \sigma^2} + \sum_{k=i}^{D} \nu_k = 0 \tag{38}$$

Using the transformation $P_i = \log(x_i)$ and setting $\nu_i = \mu_i$, we observe that any solution of (34) satisfies (38). Also, complementary slackness corresponding to (32) is satisfied if and only if it is satisfied by those for (35). Since (35) is convex, any solution satisfying the KKTs is global optimal and through the transformation $P_i = \log(x_i)$, $\mu_i = \nu_i$ is also global optimal in the original problem. The equivalent problem in (35) can be solved using any convex optimization toolbox.

V. Numerical Results

We first consider the explicit peak temperature constrained model. As shown in Fig. 2, in general the power allocation does not possess any monotonicity. The optimal power allocation is close to the minimum of the power allocation of the energy and temperature limited cases. We study the temperature limited case in Fig. 3. When temperature is strictly increasing power is strictly decreasing. When the temperature reaches the critical temperature, the power remains constant.

We then study the implicit temperature constrained model. For the general SINR case, we initialize the signomial programming problem using a feasible power allocation of $P_i = \min_i E_i$ in all slots. For the case shown in Fig. 4, the objective function takes the value 0.0895 at the the global optimal and our experiment verifies that the single condensation method also yields 0.0895. In general, we observe that the single condensation gives solutions close the global optimal. We then present the high SINR case in Fig. 5. In all cases we studied,

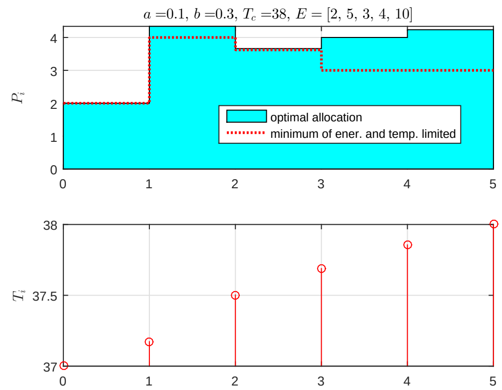

Fig. 2. Simulation for explicit temperature constraint: general case.

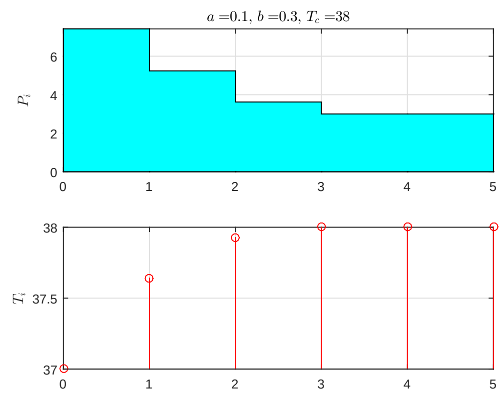

Fig. 3. Simulation for explicit temperature constraint: temperature limited.

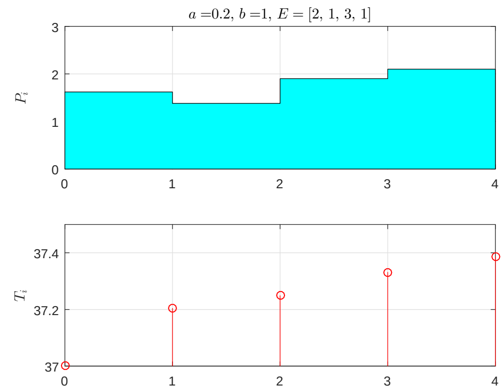

Fig. 4. Simulation for implicit temperature constraint: general case.

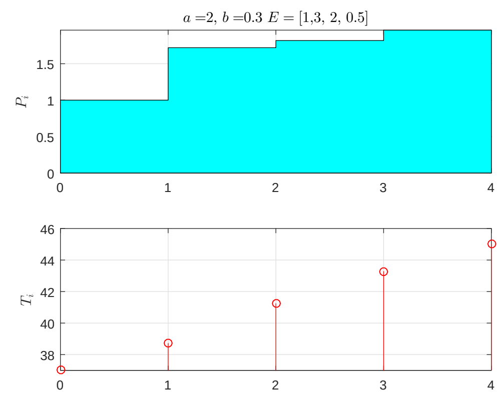

Fig. 5. Simulation for implicit temperature constraint: high SINR case.

we observe that the power allocation is non-decreasing which points to a monotonicity structure in the optimal allocation.

REFERENCES

[1] J. Yang and S. Ulukus. Optimal packet scheduling in an energy harvesting communication system. *IEEE Trans. Comm.*, 60(1):220–230, January 2012.

[2] K. Tutuncuoglu and A. Yener. Optimum transmission policies for battery limited energy harvesting nodes. *IEEE Trans. Wireless Comm.*, 11(3):1180–1189, March 2012.

[3] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. Transmission with energy harvesting nodes in fading wireless channels: Optimal policies. *IEEE JSAC*, 29(8):1732–1743, September 2011.

[4] C. K. Ho and R. Zhang. Optimal energy allocation for wireless communications with energy harvesting constraints. *IEEE Trans. Signal Proc.*, 60(9):4808–4818, September 2012.

[5] J. Yang, O. Ozel, and S. Ulukus. Broadcasting with an energy harvesting rechargeable transmitter. *IEEE Trans. Wireless Comm.*, 11(2):571–583, February 2012.

[6] J. Yang and S. Ulukus. Optimal packet scheduling in a multiple access channel with energy harvesting transmitters. *Journal of Comm. and Networks*, 14(2):140–150, April 2012.

[7] Z. Wang, V. Aggarwal, and X. Wang. Iterative dynamic water-filling for fading multiple-access channels with energy harvesting. *IEEE JSAC*, 33(3):382–395, March 2015.

[8] K. Tutuncuoglu and A. Yener. Sum-rate optimal power policies for energy harvesting transmitters in an interference channel. *Journal of Comm. Networks*, 14(2):151–161, April 2012.

[9] D. Gunduz and B. Devillers. Two-hop communication with energy harvesting. In *IEEE CAMSAP*, December 2011.

[10] C. Huang, R. Zhang, and S. Cui. Throughput maximization for the gaussian relay channel with energy harvesting constraints. *IEEE JSAC*, 31(8):1469–1479, August 2013.

[11] B. Varan and A. Yener. Delay constrained energy harvesting networks with limited energy and data storage. *IEEE JSAC*, 34(5):1550–1564, May 2016.

[12] A. Arafa, A. Baknina, and S. Ulukus. Energy harvesting two-way channels with decoding and processing costs. *IEEE Trans. on Green Comm. and Networking*, 1(1):3–16, March 2017.

[13] B. Gurakan and S. Ulukus. Cooperative diamond channel with energy harvesting nodes. *IEEE JSAC*, 34(5):1604–1617, May 2016.

[14] O. Ozel, S. Ulukus, and P. Grover. Energy harvesting transmitters that heat up: Throughput maximization under temperature constraints. *IEEE Trans. Wireless Comm.*, 15(8):5440–5452, August 2016.

[15] A. Baknina, O. Ozel, and S. Ulukus. Energy harvesting communications under temperature constraints. In *UCSD ITA*, February 2016.

[16] T. Koch, A. Lapidoth, and P. P. Sotiriadis. Channels that heat up. *IEEE Trans. Info. Theory*, 55(8):3594–3612, July 2009.

[17] D. Ward, N. Martins, and B. M. Sadler. Optimal remote estimation over action dependent switching channels: Managing workload and bias of a human operator. In *ACC*, July 2015.

[18] T. Weissman. Capacity of channels with action-dependent states. *IEEE Trans. on Info. Theory*, 56(11):5396–5411, October 2010.

[19] B. Ahmadi and O. Simeone. On channels with action-dependent states. In *IEEE ISIT*, July 2012.

[20] M. Chiang, C. W. Tan, D. P. Palomar, D. Oneill, and D. Julian. Power control by geometric programming. *IEEE Trans. Wireless Comm.*, 6(7), July 2007.

[21] P. R. Gray and R. G. Meyer. *Analysis and design of analog integrated circuits*. John Wiley & Sons Inc., 1990.

[22] B. R. Marks and G. P. Wright. A general inner approximation algorithm for nonconvex mathematical programs. *Operations Research*, 26(4):681–683, July/August 1978.

[23] M. Chiang. Geometric programming for communication systems. *Foundations and Trends in Communications and Information Theory*, 2(1–2):1–154, 2005.

[24] S. Ulukus and L. J. Greenstein. Throughput maximization in CDMA uplinks using adaptive spreading and power control. In *IEEE Sixth ISSTA*, September 2000.

[25] S. P. Boyd and L. Vandenberghe. *Convex Optimization*. Cambridge University Press, 2004.